

# Absolute Value Inequalities

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# CHAPTER 1 Absolute Value Inequalities

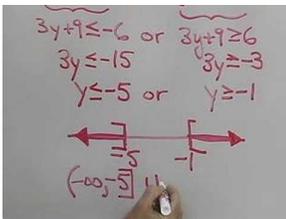
Here you'll learn about absolute value inequalities and how to find their solutions.

Suppose the perimeter of a square garden is currently  $4l$  meters, where  $l$  is the length of one of its sides, and that you want to change the perimeter to 32 meters. However, you don't want to change the perimeter by more than 8 meters. What must the length of one of its sides currently be? In this Concept, you'll learn how to solve absolute value inequalities that represent scenarios such as this one.

## Watch This

For assistance with graphing absolute value inequalities, visit this YouTube video:

[Algebra - Inequalities with Absolute Value](#)



## MEDIA

Click image to the left or use the URL below.

URL: <http://www.ck12.org/flx/render/embeddedobject/95>

Or you can also visit this link on TheMathPage: <http://www.themathpage.com/alg/absolute-value.htm> .

## Guidance

An absolute value inequality is a combination of two concepts: absolute values and linear inequalities. Therefore, to solve an absolute value inequality, you must use the problem-solving methods of each concept.

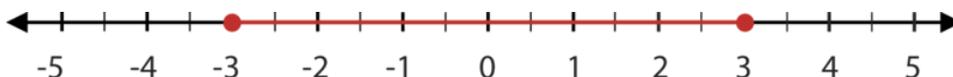
- To solve an absolute value equation, use the definition.
  - If  $d = |x - a|$ , then  $x - a = d$  OR  $x - a = -d$ .
- To solve a linear inequality, use the information learned in the previous Concepts.
  - Remember, when dividing by a negative, the inequality symbol must be reversed!

Let's begin by looking at an example.

## Example A

$$|x| \leq 3$$

Since  $|x|$  represents the distance from zero, the solutions to this inequality are those numbers whose distance from zero is less than or equal to 3. The following graph shows this solution:



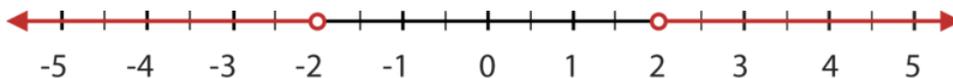
Notice that this is also the graph for the compound inequality  $-3 \leq x \leq 3$ .

Below is a second example.

**Example B**

$$|x| > 2$$

Since the absolute value of  $x$  represents the distance from zero, the solutions to this inequality are those numbers whose distances from zero are more than 2. The following graph shows this solution.



In general, the solutions to absolute value inequalities take two forms:

1. If  $|x| < a$ , then  $x < a$  or  $x > -a$ .
2. If  $|x| > a$ , then  $x > a$  or  $x < -a$ .

**Example C**

Solve  $|x + 5| > 7$ .

**Solution:**

This equation fits situation 2. Therefore,  $x + 5 > 7$  OR  $x + 5 < -7$ .

Solve each inequality separately.

$$\begin{aligned} x + 5 &> 7 \\ x &> 2 \end{aligned}$$

$$\begin{aligned} x + 5 &< -7 \\ x &< -12 \end{aligned}$$

The solutions are all values greater than two or less than  $-12$ .

**Video Review****MEDIA**

Click image to the left or use the URL below.

URL: <http://www.ck12.org/flx/render/embeddedobject/79583>

**MEDIA**

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## Guided Practice

The velocity of an object is given by the formula  $v = 25t - 80$ , where the time is expressed in seconds and the velocity is expressed in feet per second. Find the times when the velocity is greater than or equal to 60 feet per second.

### Solution:

We want to find the times when the velocity is greater than or equal to 60 feet per second. Using the formula for velocity,  $v = 25t - 80$ , and substituting the appropriate values, we obtain the absolute value inequality  $|25t - 80| \geq 60$ .

This is an example like case 2. Separate and solve.

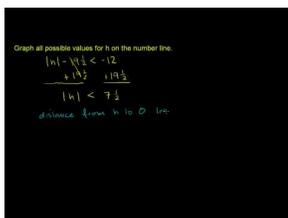
$$25t - 80 \geq 60 \text{ or } 25t - 80 \leq -60$$

$$25t \geq 140 \text{ or } 25t \leq 20$$

$$t \geq 5.6 \text{ or } t \leq 0.8$$

## Explore More

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both. [CK-12 Basic Algebra: Absolute Value Inequalities](#) (3:26)



### MEDIA

Click image to the left or use the URL below.

URL: <http://www.ck12.org/flx/render/embeddedobject/96>

1. You are asked to solve  $|a + 1| \leq 4$ . What two inequalities does this separate into?

In 2–21, solve the inequality and show the solution graph.

2.  $|x| \leq 6$
3.  $4 \leq |a + 4|$
4.  $|x| > 3.5$
5.  $6 > |10b + 6|$
6.  $|x| < 12$
7.  $|\frac{w}{10}| < 2$
8.  $|\frac{x}{5}| \leq 6$
9.  $|7x| \geq 21$
10.  $|6c + 5| < 47$
11.  $|x - 5| > 8$
12.  $|x + 7| < 3$
13.  $|x - \frac{3}{4}| \leq \frac{1}{2}$
14.  $|2x - 5| \geq 13$
15.  $|5x + 3| < 7$
16.  $|\frac{x}{3} - 4| \leq 2$
17.  $|\frac{2x}{7} + 9| > \frac{5}{7}$
18.  $|-6t + 3| + 9 \geq 18$

19.  $|9p + 5| > 23$
20.  $|-2s - 4| \leq 6$
21.  $\frac{|10m - 5|}{8} > 5$
22. A three-month-old baby boy weighs an average of 13 pounds. He is considered healthy if he is 2.5 pounds more or less than the average weight. Find the weight range that is considered healthy for three-month-old boys.

### Mixed Review

23. Graph  $y = 3|x - 3|$ .
24. Graph  $y = \frac{5}{4}x - 2$ .
25. What is 14.75% of 29?
26. A shirt was sold for \$31.99 after a 15% markup. What was the original price?
27. Using the formula to convert Celsius to Fahrenheit found in a previous Concept, determine the Fahrenheit equivalent to  $16^{\circ}\text{C}$ .
28. Charlene has 18 more apples than Raul. Raul has 36 apples. Write an equation to represent this situation and determine how many apples Charlene has.
29. Suppose you flip a coin. What are the possible outcomes?
30. How many degrees are in a circle?