Absolute Value Inequalities

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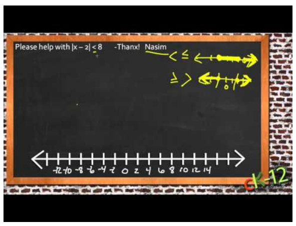
CHAPTER 1 Absolute Value Inequalities

Here you'll learn how to solve absolute value inequalities and show their solution graph. You'll also solve real-world problems involving absolute value inequalities.

What if you were given an absolute value inequality like $|2x| \leq 16$? How could you solve it? After completing this Concept, you'll be able to find the solution set and show the solution graph of inequalities like this one.

Watch This



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CK-12 Foundation: 0610S Absolute Value Inequalities (H264)

Guidance

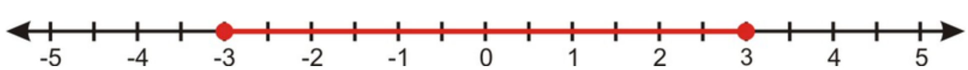
Absolute value inequalities are solved in a similar way to absolute value equations. In both cases, you must consider the same two options:

1. The expression inside the absolute value is not negative.
2. The expression inside the absolute value is negative.

Then you must solve each inequality separately.

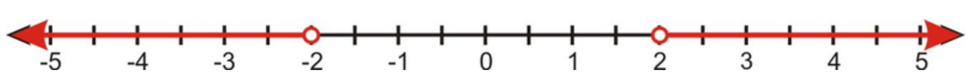
Solve Absolute Value Inequalities

Consider the inequality $|x| \leq 3$. Since the absolute value of x represents the distance from zero, the solutions to this inequality are those numbers whose distance from zero is less than or equal to 3. The following graph shows this solution:



Notice that this is also the graph for the compound inequality $-3 \leq x \leq 3$.

Now consider the inequality $|x| > 2$. Since the absolute value of x represents the distance from zero, the solutions to this inequality are those numbers whose distance from zero are more than 2. The following graph shows this solution.



Notice that this is also the graph for the compound inequality $x < -2$ or $x > 2$.

Example A

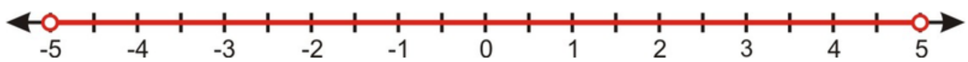
Solve the following inequalities and show the solution graph.

a) $|x| < 5$

b) $|x| \geq 2.5$

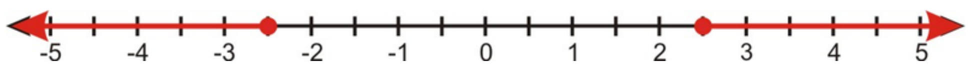
Solution

a) $|x| < 5$ represents all numbers whose distance from zero is less than 5.



This answer can be written as "$-5 < x < 5$".

b) $|x| \geq 2.5$ represents all numbers whose distance from zero is more than or equal to 2.5



This answer can be written as "$x \leq -2.5$ or $x \geq 2.5$".

Rewrite and Solve Absolute Value Inequalities as Compound Inequalities

In the last section you saw that absolute value inequalities are compound inequalities.

Inequalities of the type $|x| < a$ can be rewritten as "$-a < x < a$".

Inequalities of the type $|x| > b$ can be rewritten as "$x < -b$ or $x > b$."

To solve an absolute value inequality, we separate the expression into two inequalities and solve each of them individually.

Example B

Solve the inequality $|x-3| < 7$ and show the solution graph.

Solution

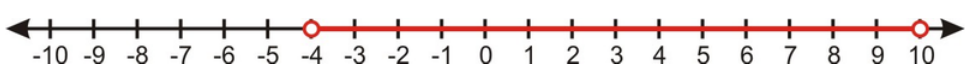
Re-write as a compound inequality: $-7 < x-3 < 7$

Write as two separate inequalities: $x-3 < 7$ and $x-3 > -7$

Solve each inequality: $x < 10$ and $x > -4$

Re-write solution: $-4 < x < 10$

The solution graph is



We can think of the question being asked here as "What numbers are within 7 units of 3?"; the answer can then be expressed as "All the numbers between -4 and 10."

Example C

Solve the inequality $|4x+5| \leq 13$ and show the solution graph.

Solution

Re-write as a compound inequality: $-13 \leq 4x+5 \leq 13$

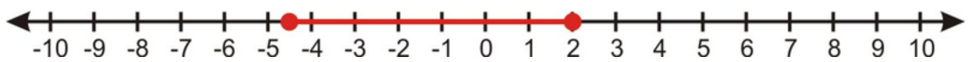
Write as two separate inequalities: $4x+5 \leq 13$ and $4x+5 \geq -13$

Solve each inequality: $4x \leq 8$ and $4x \geq -18$

$x \leq 2$ and $x \geq -\frac{9}{2}$

Re-write solution: $-\frac{9}{2} \leq x \leq 2$

The solution graph is



Example D

Solve the inequality $|x+12| > 2$ *and show the solution graph.*

Solution

Re-write as a compound inequality: $x+12 < -2$ or $x+12 > 2$

Solve each inequality: $x < -14$ or $x > -10$

The solution graph is

Solve Real-World Problems Using Absolute Value Inequalities

Absolute value inequalities are useful in problems where we are dealing with a range of values.

Example E

The velocity of an object is given by the formula $v = 25t - 80$*, where the time is expressed in seconds and the velocity is expressed in feet per second. Find the times for which the magnitude of the velocity is greater than or equal to 60 feet per second.*

Solution

The *magnitude* of the velocity is the absolute value of the velocity. If the velocity is $25t-80$ feet per second, then its magnitude is $|25t-80|$ feet per second. We want to find out when that magnitude is greater than or equal to 60, so we need to solve $|25t-80| \geq 60$ for t.

First we have to split it up: $25t-80 \geq 60$ or $25t-80 \leq -60$

Then solve: $25t \geq 140$ or $25t \leq 20$

$t \geq 5.6$ or $t \leq 0.8$

The magnitude of the velocity is greater than 60 ft/sec for times **less than 0.8 seconds** and for times **greater than 5.6 seconds.**

When $t = 0.8\ seconds,\ v = 25(0.8) - 80 = -60\ ft/sec$. The magnitude of the velocity is 60 ft/sec. (The negative sign in the answer means that the object is moving backwards.)

When $t = 5.6\ seconds,\ v = 25(5.6) - 80 = 60\ ft/sec$.

To find where the magnitude of the velocity is **greater** than 60 ft/sec, check some arbitrary values in each of the following time intervals: $t \leq 0.8,\ 0.8 \leq t \leq 5.6$ and $t \geq 5.6$.

Check $t = 0.5: \ v = 25(0.5) - 80 = -67.5 \ ft/sec$

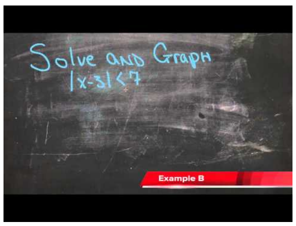
Check $t = 2: \ v = 25(2) - 80 = -30 \ ft/sec$

Check $t = 6: \ v = 25(6) - 80 = -70 \ ft/sec$

You can see that the magnitude of the velocity is greater than 60 ft/sec only when $t \geq 5.6$ or when $t \leq 0.8$.

The answer checks out.

Watch this video for help with the Examples above.



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CK-12 Foundation: Absolute Value Inequalities

Vocabulary

- The absolute value of a number is its distance from zero on a number line.
- $|x| = x$ if x is not negative, and $|x| = -x$ if x is negative.
- An equation or inequality with an absolute value in it **splits into two equations,** one where the expression inside the absolute value sign is positive and one where it is negative. When the expression within the absolute value is **positive**, then the absolute value signs do nothing and can be omitted. When the expression within the absolute value is **negative,** then the expression within the absolute value signs must be negated before removing the signs.
- Inequalities of the type $|x| < a$ can be rewritten as "$-a < x < a$."
- Inequalities of the type $|x| > b$ can be rewritten as "$x < -b$ or $x > b$."

Guided Practice

Solve the inequality $|8x - 15| \geq 9$ *and show the solution graph.*

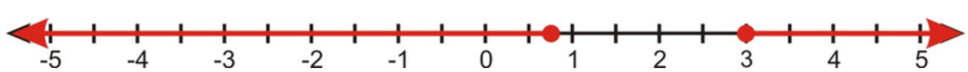
Solution

Re-write as a compound inequality: $8x - 15 \leq -9$ or $8x - 15 \geq 9$

Solve each inequality: $8x \leq 6$ or $8x \geq 24$

$x \leq \frac{3}{4}$ or $x \geq 3$

The solution graph is



Explore More

Solve the following inequalities and show the solution graph.

1. $|x| \leq 6$

2. $|x| > 3.5$
3. $|x| < 12$
4. $|x| > 10$
5. $|7x| \geq 21$
6. $|x-5| > 8$
7. $|x+7| < 3$
8. $\left|x-\frac{3}{4}\right| \leq \frac{1}{2}$
9. $|2x-5| \geq 13$
10. $|5x+3| < 7$
11. $\left|\frac{x}{3}-4\right| \leq 2$
12. $\left|\frac{2x}{7}+9\right| > \frac{5}{7}$
13.
    a. How many solutions does the inequality $|x| \leq 0$ have?
    b. How about the inequality $|x| \geq 0$?
14. A company manufactures rulers. Their 12-inch rulers pass quality control if they are within $\frac{1}{32}$ *inches* of the ideal length. What is the longest and shortest ruler that can leave the factory?
15. A three month old baby boy weighs an average of 13 pounds. He is considered healthy if he is at most 2.5 lbs. more or less than the average weight. Find the weight range that is considered healthy for three month old boys.