Solving Absolute Value Inequalities

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CHAPTER 1 Solving Absolute Value Inequalities

Here you'll learn how to solve absolute value inequalities.

The tolerance for the weight of a volleyball is 2.6 grams. If the average volleyball weighs 260 grams, what is the range of weights for a volleyball?

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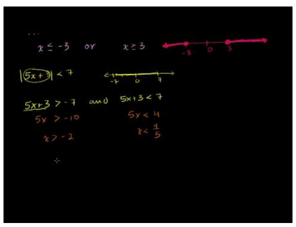

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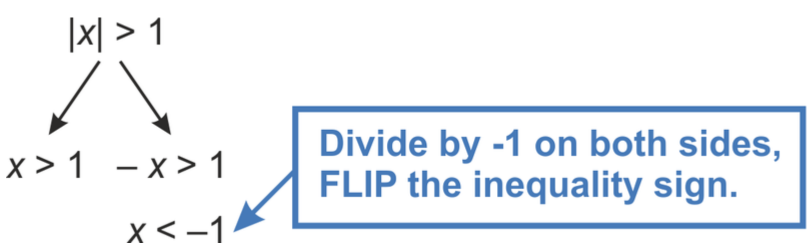
Khan Academy: Absolute Value Inequalities

Guidance

Like absolute value equations, absolute value inequalities also will have two answers. However, they will have a range of answers, just like compound inequalities.

$|x| > 1$ This inequality will have two answers, when x is 1 and when $-x$ is 1. But, what about the inequality sign? The two possibilities would be:



Notice in the second inequality, we did not write $x > -1$. This is because what is inside the absolute value sign can be positive or negative. Therefore, if x is negative, then $-x > 1$. It is a very important difference between the two inequalities. Therefore, for the first solution, we leave the inequality sign the same and for the second solution we need to change the sign of the answer AND flip the inequality sign.

Example A

Solve $|x+2| \leq 10$.

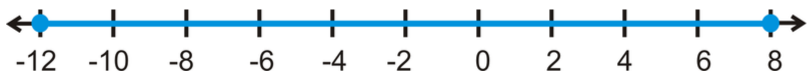
Solution: There will be two solutions, one with the answer and sign unchanged and the other with the inequality sign flipped and the answer with the opposite sign.

$$\begin{array}{cc} & |x+2| \leq 10 \\ & \swarrow \searrow \\ x+2 \leq 10 & x+2 \geq -10 \\ x \leq 8 & x \geq -12 \end{array}$$

Test a solution, $x = 0$:

$$\begin{aligned} |0+2| &\leq 10 \\ |2| &\leq 10 \end{aligned}$$

When graphing this inequality, we have



Notice that this particular absolute value inequality has a solution that is an "and" inequality because the solution is between two numbers.

If $|ax+b| < c$, then $-c < ax+b < c$.

If $|ax+b| \leq c$, then $-c \leq ax+b \leq c$.

If $|ax+b| > c$, then $ax+b < -c$ or $ax+b > c$.

If $|ax+b| \geq c$, then $ax+b \leq -c$ or $ax+b \geq c$.

If you are ever confused by the rules above, always test one or two solutions and graph it.

Example B

Solve and graph $|4x-3| > 9$.

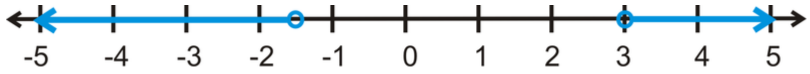
Solution: Break apart the absolute value inequality to find the two solutions.

$$\begin{array}{cc} & |4x-3| > 9 \\ & \swarrow \searrow \\ 4x-3 > 9 & 4x-3 < -9 \\ 4x > 12 & 4x < -6 \\ x > 3 & x < -\frac{3}{2} \end{array}$$

Test a solution, $x = 5$:

$$\begin{aligned} |4(5)-3| &> 9 \\ |20-3| &> 9 \\ 17 &> 9 \end{aligned}$$

The graph is:



Example C

Solve $|-2x+5| < 11$.

Solution: Given the rules above, this will become an "and" inequality.

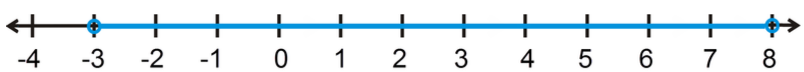
$$|-2x+5| < 11$$

$$\swarrow \searrow$$

$$\begin{aligned} -2x+5 &< 11 & -2x+5 &> -11 \\ -2x &< 6 & -2x &> -16 \\ x &> -3 & x &< 8 \end{aligned}$$

The solution is x is greater than -3 and less than 8. In other words, the solution is $-3 < x < 8$.

The graph is:



Intro Problem Revisit Set up an absolute value inequality. w is the range of weights of the volleyball.

$$|w-260| \leq 2.6$$

$$\swarrow \searrow$$

$$\begin{aligned} w-260 &\leq 2.6 & w-260 &\geq -2.6 \\ w &\leq 262.6 & w &\geq 257.4 \end{aligned}$$

So, the range of the weight of a volleyball is $257.4 \leq w \leq 262.6$ grams.

Guided Practice

1. Is $x = -4$ a solution to $|15-2x| > 9$?

2. Solve and graph $\left|\frac{2}{3}x+5\right| \leq 17$.

Answers

1. Plug in -4 for x to see if it works.

$$\begin{aligned} |15-2(-4)| &> 9 \\ |15+8| &> 9 \\ |23| &> 9 \\ 23 &> 9 \end{aligned}$$

Yes, -4 works, so it is a solution to this absolute value inequality.

2. Split apart the inequality to find the two answers.

$$\left|\frac{2}{3}x+5\right| \leq 17$$

$$\swarrow \quad \searrow$$

$$\begin{aligned} \left|\frac{2}{3}x+5\right| &\leq 17 & \frac{2}{3}x+5 &\geq -17 \\ \frac{2}{3}x &\leq 12 & \frac{2}{3}x &\geq -22 \\ x &\leq 12 \cdot \frac{3}{2} & x &\geq -22 \cdot \frac{3}{2} \\ x &\leq 18 & x &\geq -33 \end{aligned}$$

Test a solution, $x = 0$:

$$\begin{aligned} \left|\frac{2}{3}(0)+5\right| &\leq 17 \\ |5| &\leq 17 \\ 5 &\leq 17 \end{aligned}$$

Explore More

Determine if the following numbers are solutions to the given absolute value inequalities.

1. $|x-9| > 4; 10$
2. $\left|\frac{1}{2}x-5\right| \leq 1; 8$
3. $|5x+14| \geq 29; -8$

Solve and graph the following absolute value inequalities.

4. $|x+6| > 12$
5. $|9-x| \leq 16$
6. $|2x-7| \geq 3$
7. $|8x-5| < 27$
8. $\left|\frac{5}{6}x+1\right| > 6$
9. $|18-4x| \leq 2$
10. $\left|\frac{3}{4}x-8\right| > 13$
11. $|6-7x| \leq 34$
12. $|19+3x| \geq 46$

Solve the following absolute value inequalities. a is greater than zero.

13. $|x-a| > a$
14. $|x+a| \leq a$
15. $|a-x| \leq a$