Algebraic Solutions to Absolute Value Inequalities

Brenda Meery
Kaitlyn Spong

To access a customizable version of this book, as well as other interactive content, visit www.ck12.org

AUTHORS
Brenda Meery
Kaitlyn Spong

CK-12 Foundation is a non-profit organization with a mission to reduce the cost of textbook materials for the K-12 market both in the U.S. and worldwide. Using an open-source, collaborative, and web-based compilation model, CK-12 pioneers and promotes the creation and distribution of high-quality, adaptive online textbooks that can be mixed, modified and printed (i.e., the FlexBook® textbooks).

Printed: March 30, 2015

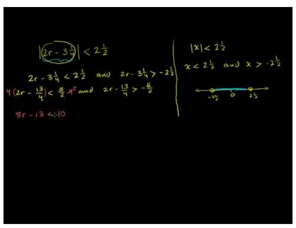
CHAPTER 1 Algebraic Solutions to Absolute Value Inequalities

Here you'll learn how to solve an absolute value linear inequality.

A ball is fired from the cannon during the Independence Day celebrations. It is fired directly into the air with an initial velocity of 150 ft/sec. The speed of the cannon ball can be calculated using the formula $s = |-32t + 150|$, where s is the speed measure in ft/sec and t is the time in seconds. Calculate the times when the speed is less than 86 ft/sec.

Watch This

Khan Academy Absolute Value Inequalities



MEDIA

Click image to the left or use the URL below.

URL: http://www.ck12.org/flx/render/embeddedobject/58476

Guidance

You have learned that a linear inequality is of the form $ax+b > c, ax+b < c, ax+b \geq c$, or $ax+b \leq c$. Linear inequalities, unlike linear equations, have more than one solution. They have a solution set. For example, if you look at the linear inequality $x+3 > 5$. You know that $2+3$ is equal to 5, therefore the solution set could be any number greater than 2.

Recall that when solving absolute value linear equations, you have to solve for the two related equations. Remember that for $|ax+b| = c$, you had to solve for $ax+b = c$ and $ax+b = -c$. The same is true for linear inequalities. If you have an absolute value linear inequality, you would need to solve for the two related linear inequalities.

The table below shows the four types of absolute value linear inequalities and the two related inequalities required to be solved for each one.

TABLE 1.1:

Absolute Value Inequality	$\vert ax+b\vert > c$	$\vert ax+b\vert < c$	$\vert ax+b\vert \geq c$	$\vert ax+b\vert \leq c$
Inequality 1	$ax+b > c$	$ax+b < c$	$ax+b \geq c$	$ax+b \leq c$
Inequality 2	$ax+b < -c$	$ax+b > -c$	$ax+b \leq -c$	$ax+b \geq -c$

Remember the rules to algebraically solve for the variable remain the same as you have used before.

Example A

Solve for the absolute value inequality $|g+5| < 3$.

Solution: Set up and solve two inequalities:

$$\begin{aligned} g+5 &< 3 \\ g+5-5 &< 3-5 && \text{Subtract 5 from both sides to isolate the variable} \\ g &< -2 \\ &OR \\ g+5 &> -3 \\ g+5-5 &> -3-5 && \text{Subtract 5 from both sides to isolate the variable} \\ g &> -8 \end{aligned}$$

Solution: $-8 < g < -2$

Example B

Solve for the absolute value inequality $|j-\frac{1}{2}| > 2$.

Solution: Set up and solve two inequalities:

$$\begin{aligned} j-\frac{1}{2} &> 2 \\ \left(\frac{2}{2}\right) j-\frac{1}{2} &> \left(\frac{2}{2}\right) 2 && \text{Multiply to get a common denominator (LCD} = 2) \\ \frac{2j}{2}-\frac{1}{2} &> \frac{2}{2} && \text{Simplify} \\ 2j-1 &> 2 && \text{Simplify} \\ 2j-1+1 &> 2+1 && \text{Add 1 to both sides to isolate the variable} \\ 2j &> 3 && \text{Simplify} \\ \frac{2j}{2} &> \frac{3}{2} && \text{Divide by 2 to solve for the variable.} \\ j &> \frac{3}{2} \\ &OR \\ j-\frac{1}{2} &< -2 \\ \left(\frac{2}{2}\right) j-\frac{1}{2} &< \left(\frac{2}{2}\right)(-2) && \text{Multiply to get a common denominator (LCD} = 2) \\ \frac{2j}{2}-\frac{1}{2} &< \frac{-2}{2} && \text{Simplify} \\ 2j-1 &< -2 && \text{Simplify} \\ 2j-1+1 &< -2+1 && \text{Add 1 to both sides to isolate the variable} \\ 2j &< -1 && \text{Simplify} \\ \frac{2j}{2} &< \frac{-1}{2} && \text{Divide by 2 to solve for the variable.} \\ j &< \frac{-1}{2} \end{aligned}$$

Solution: $j > \frac{3}{2}$ or $j < \frac{-1}{2}$

Example C

Solve for the absolute value inequality $|t+1|-3 \geq 2$.

Solution: First, isolate the absolute value part of the inequality:

$$\begin{aligned}
|t+1|-3 &\geq 2 \\
|t+1|-3+3 &\geq 2+3 \\
|t+1| &\geq 5
\end{aligned}$$

Now, set up and solve the two inequalities:

$$\begin{aligned}
t+1 &\geq 5 \\
t+1-1 &\geq 5-1 \\
t &\geq 4 \\
&OR \\
t+1 &\leq -5 \\
t+1-1 &\leq -5-1 \\
t &\leq -6
\end{aligned}$$

Solution: $t \geq 4$ or $t \leq -6$.

Concept Problem Revisited

A ball is fired from the cannon during the Independence Day celebrations. It is fired directly into the air with an initial velocity of 150 ft/sec. The speed of the cannon ball can be calculated using the formula $s = |-32t+150|$, where s is the speed measure in ft/sec and t is the time in seconds. Calculate the times when the speed is less than 86 ft/sec.

$86 > |-32t+150|$

$$\begin{aligned}
86 &> -32t+150 \\
86-150 &> -32t+150-150 && \text{Subtract 150 from both sides to isolate the variable} \\
-64 &> -32t && \text{Simplify} \\
\frac{-64}{-32} &< \frac{-32t}{-32} && \text{Divide by -32 to solve for the variable. Remember when} \\
& && \quad \text{dividing by a negative number to reverse the sign of the inequality.} \\
t &> 2 \\
& OR \\
-86 &< -32t+150 \\
-86-150 &< -32t+150-150 && \text{Subtract 150 from both sides to isolate the variable} \\
-236 &< -32t && \text{Simplify} \\
\frac{-236}{-32} &> \frac{-32t}{-32} && \text{Divide by -32 to solve for the variable. Remember when} \\
& && \quad \text{dividing by a negative number to reverse the sign of the inequality.} \\
t &< 7.375
\end{aligned}$$

Therefore when $2 < t < 7.375$, the speed is greater than 86 ft/sec.

Guided Practice

Solve each inequality:

1. $|x-1| \geq 9$

2. $|-2w+7| < 23$

3. $|-4+2b|+3 \leq 21$

Answers:

1. $|x-1| \geq 9$

$$\begin{aligned}
x-1 &\geq 9 \\
x-1+1 &\geq 9+1 && \text{(Add 1 to both sides to isolate and solve for the variable)} \\
x &\geq 10 \\
& OR \\
x-1 &\leq -9 \\
x-1+1 &\leq -9+1 && \text{(Add 1 to both sides to isolate and solve for the variable)} \\
x &\leq -8
\end{aligned}$$

Solution: $x \geq 10$ or $x \leq -8$.

2. $|-2w+7| < 23$

$$\begin{aligned}
-2w+7 &< 23 \\
-2w+7-7 &< 23-7 && \text{(Subtract 7 from both sides to get variables on same side)} \\
-2w &< 16 && \text{(Simplify)} \\
\frac{-2w}{-2} &> \frac{16}{-2} && \text{(Divide by -2 to solve for the variable, reverse sign of inequality)} \\
w &> -8 \\
&OR \\
-2w+7 &> -23 \\
-2w+7-7 &> -23-7 && \text{(Subtract 7 from both sides to get variables on same side)} \\
-2w &> -30 && \text{(Simplify)} \\
\frac{-2w}{-2} &< \frac{-30}{-2} && \text{(Divide by -2 to solve for the variable, reverse sign of inequality)} \\
w &< 15
\end{aligned}$$

Solution: $-8 < w < 15$

3. First, isolate the absolute value part of the inequality:

$$\begin{aligned}
|-4+2b|+3 &\leq 21 \\
|-4+2b|+3-3 &\leq 21-3 \\
|-4+2b| &\leq 18
\end{aligned}$$

Now, set up and solve the two inequalities:

$$\begin{aligned}
-4+2b &\leq 18 \\
-4+2b+4 &\leq 18+4 \\
2b &\leq 22 \\
b &\leq 11 \\
&OR \\
-4+2b &\geq -18 \\
-4+2b+4 &\geq -18+4 \\
2b &\geq -14 \\
b &\geq -7
\end{aligned}$$

Solution: $-7 \leq b \leq 11$

Explore More

Solve each of the following absolute value linear inequalities:

1. $|p-16| > 10$
2. $|r+2| < 5$
3. $|3-2k| \geq 1$

4. $|8-y| > 5$
5. $8 \geq |5d-2|$
6. $|s+2|-5 > 8$
7. $|10+8w|-2 < 16$
8. $|2q+1|-5 \leq 7$
9. $|\frac{1}{3}(g-2)| < 4$
10. $|-2(e+4)| > 17$
11. $|-5x-3(2x-1)| > 3$
12. $|2(a-1.2)| \geq 5.6$
13. $|-2(r+3.1)| \leq 1.4$
14. $|\frac{3}{4}(m-3)| \leq 8$
15. $|-2\left(e-\frac{3}{4}\right)| \geq 3$