Absolute Value: Equations & Inequalities

Big Picture

Absolute value equations and inequalities are very similar to linear equations and inequalities. In both cases, the goal is to solve for a variable. However, unlike linear equations and linear inequalities, the variable is not a specific number. Instead, the variable represents a specific distance from zero. When solving absolute value equations and inequalities, two options need to be considered: when the expression inside the absolute value is not negative and when the expression inside the absolute value is negative.

Key Terms

Absolute Value: The absolute value of a number is the distance of that number from 0.

Absolute Value Equations

An **absolute value** equation is an equation that contains an absolute value expression

Example: $|ax+b| = c$, where $c \geq 0$

To solve an absolute value equation, split it into two equations and solve individually.

Example: $|ax+b| = c$, where $c \geq 0$

- Split into two equations: $ax+b = c$ and $ax+b = -c$
- Solve: $x = \frac{c-b}{a}$ and $x = \frac{c+b}{a}$ are both solutions

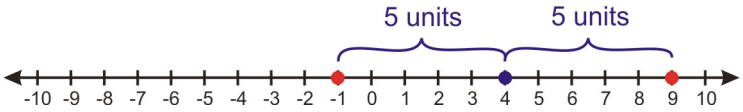
Do not start to solve until the absolute value equation is split into two equations.

Example: $|x-4| = 5$

$x-4 = 5$ and $x-4 = -5$

$x = 9$ $x = -1$

Plotted on the number line:



- The two solutions are both 5 units away from 4

Absolute Value Inequalities

An absolute value inequality is an inequality that contains an absolute value expression

To solve an absolute value inequality, split into two inequalities, and solve individually.

Example: $|ax + b| < c$, where $c \geq 0$

- Split into two inequalities: $ax + b > -c$ and $ax + b < c$ (can be rewritten as $-c < ax + b < c$)
  - Absolute value inequalities can be rewritten as compound inequalities with "and"
- Solve: $x > -\frac{c+b}{a}$ and $x < \frac{c-b}{a}$ (can be rewritten as $-\frac{c+b}{a} < x < \frac{c-b}{a}$)

Example: $|ax + b| > c$, where $c \geq 0$

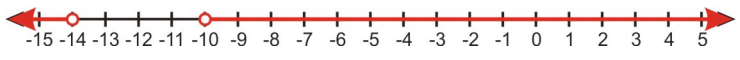
- Split into two inequalities: $ax + b < -c$ or $ax + b > c$
  - Absolute value inequalities can be rewritten as compound inequalities with "or"
- Solve: $x < -\frac{c+b}{a}$ or $x > \frac{c-b}{a}$

If the sign is less thAN, then it's a compound inequality with AND. If the sign is greatER than, then it's a compound equality with OR. Remember: less thAND, greatOR.

Example: $|x + 12| > 2$

$x + 12 < -2$ or $x + 12 > 2$

$x < -14$ $x > -10$



Example: $|4x + 5| \leq 13$

$4x + 5 \geq -13$ and $4x + 5 \leq 13$

$4x \geq -18$ $4x \leq 8$

$x \geq -\frac{9}{2}$ $x \leq 2$

Notes