

Using the General Absolute Value Equation and the Graphing Calculator

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CHAPTER

1

Using the General Absolute Value Equation and the Graphing Calculator

Here you'll learn how to graph more complicated absolute value functions and use the graphing calculator.

Mrs. Patel assigns the absolute value function $y = -|x + 3| - 2$. She tells her students to find the vertex.

"This is hard," George laments. "I'm going to need a calculator."

"No, it's not," Sarai counters. "I can tell you what the vertex is without even graphing by hand."

Who is right and what is the vertex?

Guidance

In the problem set of the previous concept, we were introduced to the general equation of an absolute value function. Let's formally define it here.

General Form of an Absolute Value Function: For any absolute value function, the general form is $y = a|x - h| + k$, where a controls the width of the "V" and (h, k) is the vertex.

You probably made these connections during the problem set from the previous concept. Now, we will put it all to use together.

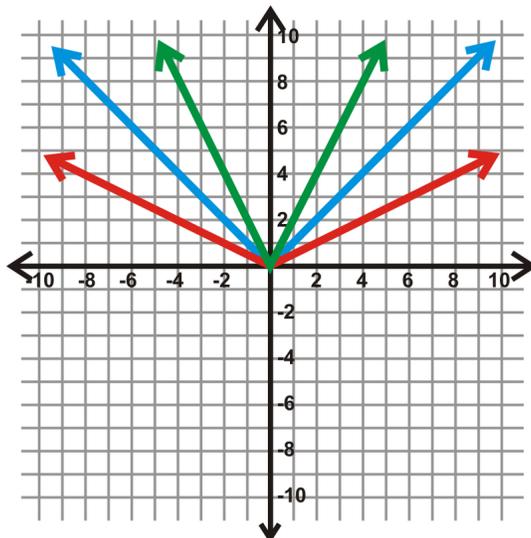
Example A

Graph $y = |x|$, $y = \frac{1}{2}|x|$, and $y = 2|x|$ on the same set of axes. Compare the three functions.

Solution: You can make a table for all three of these functions. However, now that we have a better understanding of absolute value functions, let's use some patterns. First, look at the vertex. Nothing is being added or subtracted, so the vertex for all three will be $(0, 0)$. Second, look at " a ." For an absolute value function, we can think of a like the slope. Referring back to the definition of the parent graph, each function above can be rewritten as:

$$y = \begin{cases} x; x \geq 0 \\ -x; x < 0 \end{cases} \quad (\text{blue}), \quad y = \begin{cases} \frac{1}{2}x; x \geq 0 \\ -\frac{1}{2}x; x < 0 \end{cases} \quad (\text{red}), \quad \text{and} \quad y = \begin{cases} 2x; x \geq 0 \\ -2x; x < 0 \end{cases} \quad (\text{green})$$

Comparing the three, we see that if the slope is between 1 and 0, the opening is wider than the parent graph. If the slope, or a , is greater than 1, then that opening is narrower. The amount of the opening between the two sides of an absolute value function (and other functions) is called the **breadth**.

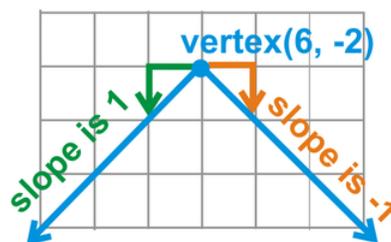
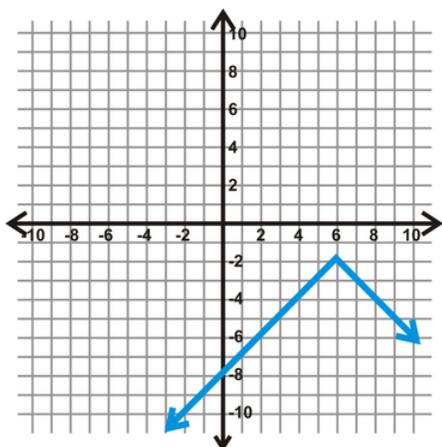


Now, in addition to drawing a table, we can use the general form of an absolute value equation and the value of a to find the shape of the V .

Example B

Without making a table, sketch the graph of $y = -|x - 6| - 2$.

Solution: First, determine the vertex. From the general form, we know that it will be $(6, -2)$. Notice that the x -variable is the opposite sign of what is in the equation; the y -variable is the same. That is our starting point. Then, we have a negative sign in front of the absolute value. This means our V will open down. Finally, there is no a term, so we can assume it is 1, meaning that the slope of each side of the V will be 1 and -1 .



Lastly, we can use a graphing calculator to help us graph absolute value equations. The directions given here pertain to the TI-83/84 series; however every graphing calculator should be able to graph absolute value functions.

Example C

Use a graphing calculator to graph $y = |4x + 1| - 2$. Find the vertex, domain, and range.

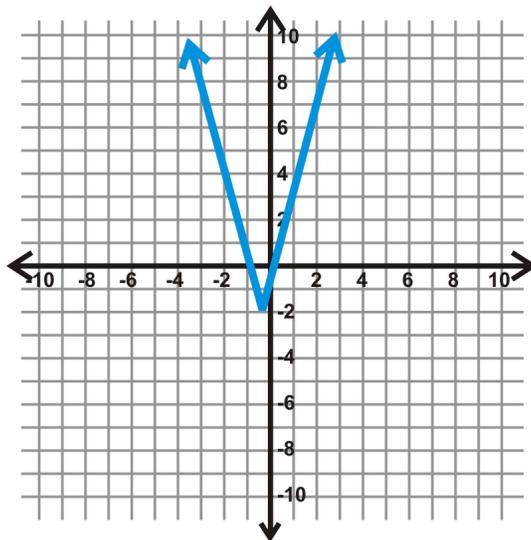
Solution: For the TI-83/84

1. Press the $Y =$ button.

- Clear any previous functions (press CLEAR) and turn off any previous plots (arrow up to Plot 1 and press ENTER).
- Press the MATH button, arrow over to NUM and highlight **1:abs**(. Press ENTER.
- Type in the remaining portion of the function. The screen:

Plot1 Plot2 Plot3
 $\backslash Y_1 = \text{abs}(4X+1) - 2$

- Press GRAPH. If your screen is off, press ZOOM, scroll down to **6:ZStandard**, and press ENTER. The graph looks like:



As you can see from the graph, the vertex is not $(-1, -2)$. The y -coordinate is -2 , but the 4 inside the absolute value affects the x -coordinate. Set what is inside the absolute value equal to zero to solve for the x -coordinate of the vertex.

$$\begin{aligned} 4x + 1 &= 0 \\ 4x &= -1 \\ x &= -\frac{1}{4} \end{aligned}$$

The vertex is $(-\frac{1}{4}, -2)$. From the previous concept, we know that the domain is all real numbers. The range will be any number greater than and including -2 . In this function, the “ a ” term was inside the absolute value. When this happens, it will always affect the x -coordinate of the vertex.

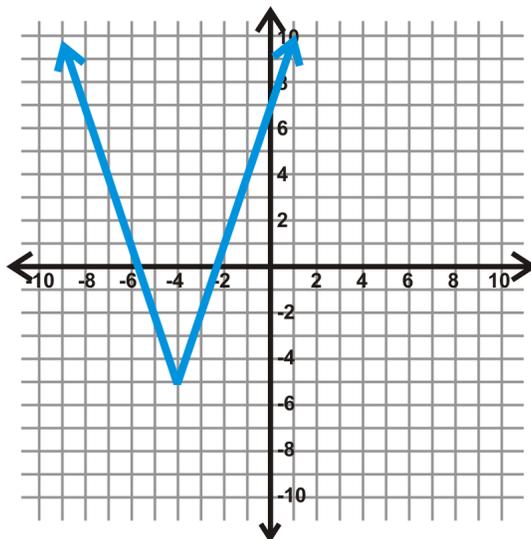
Intro Problem Revisit Sarai is right. The absolute value function is written in general form, so a calculator is not necessary. The vertex is $(-3, -2)$.

Guided Practice

- Graph $y = 3|x + 4| - 5$ without a graphing calculator or making a table. Find the vertex, domain, and range of the function.
- Graph $y = -2|x - 5| + 1$ using a graphing calculator.

Answers

1. First, use the general form to find the vertex, $(-4, -5)$. Then, use a to determine the breadth of the function. $a = 3$, so we will move up 3 and over 1 *in both directions* to find the points on either side of the vertex.

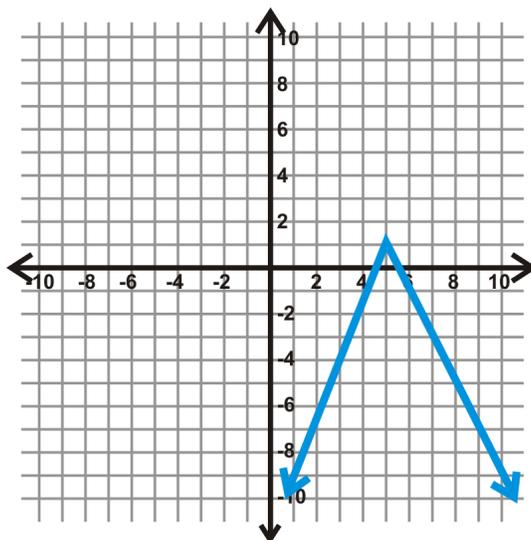


The domain is all real numbers and the range is all reals greater than and including -5 .

Domain: $x \in \mathbb{R}$

Range: $y \in [-5, \infty)$

2. Using the steps from Example C, the function looks like:



Explore More

- Graph $y = 3|x|$, $y = -3|x|$, and $y = |-3x|$ on the same set of axes. Compare the graphs.
- Graph $y = \frac{1}{4}|x+1|$, and $y = \frac{1}{4}|x|+1$ on the same set of axes. Compare the graphs.
- Without graphing, do you think that $y = 2|x|$, $y = |2x|$, and $y = |-2x|$ will all produce the same graph? Why or why not?

4. We know that the domain of all absolute value functions is all real numbers. What would be a general rule for the range?

Use the general form and pattern recognition to graph the following functions. Determine the vertex, domain, and range. No graphing calculators!

5. $y = |x - 2| + 5$
6. $y = -2|x + 3|$
7. $y = \frac{1}{3}|x| + 4$
8. $y = 2|x + 1| - 2$
9. $y = -\frac{1}{2}|x - 7|$
10. $y = -|x - 8| + 6$

Use a graphing calculator to graph the following functions. Sketch a copy of the graph on your paper. Identify the vertex, domain, and range.

11. $y = -4|2x + 1|$
12. $y = \frac{2}{3}|x - 4| + \frac{1}{2}$
13. $y = \frac{4}{3}|2x - 3| - \frac{7}{2}$

Graphing Calculator Extension Use the graphing calculator to answer questions 14-16.

14. Graph $y = x^2 - 4$ on your calculator. Sketch the graph and determine the domain and range.
15. Graph $y = |x^2 - 4|$ on your calculator. Sketch graph and determine the domain and range.
16. How do the two graphs compare? How are they different? What could you do to the first graph to get the second?