Graphical Solutions to Absolute Value Inequalities

Brenda Meery
Kaitlyn Spong

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AUTHORS
Brenda Meery
Kaitlyn Spong

CHAPTER 1

Graphical Solutions to Absolute Value Inequalities

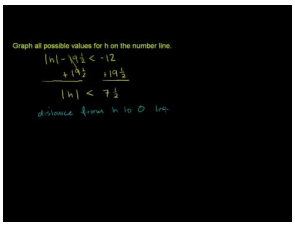
Here you'll learn how to represent the solutions of an absolute value inequality on a number line.

Solve the following inequality and graph the solution on a number line.

$|x+2| \leq 3$

Watch This

Khan Academy Absolute Value Inequalities on a Number Line



MEDIA

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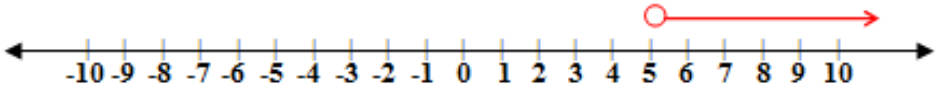
Guidance

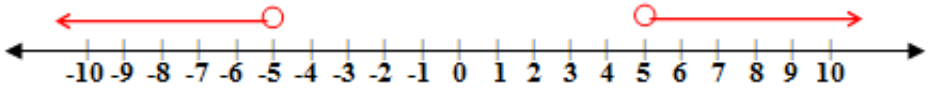
Recall that you can graph linear inequalities on number lines. For $x > 5$, the graph can be shown as:



Notice that there is only one solution set and therefore one section of the number line has the region shown in red.

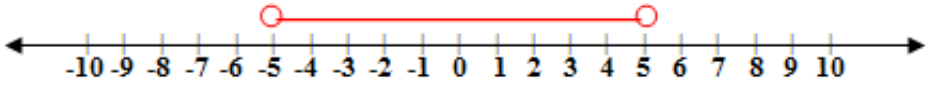
What do you think would happen with absolute value linear inequalities? With absolute value linear inequalities, there are two inequalities to solve. Therefore there can be two sections of the number line showing solutions.

For $|t| > 5$, you would actually solve for $t > 5$ and $t < -5$. If you were to graph this solution on a number line it would look like the following:



The solution is $t > 5$ OR $t < -5$.

For $|t| < 5$, you would actually solve for $t < 5$ and $t > -5$. If you were to graph this solution on a number line it would look like the following:



The solution is $-5 < t < 5$. This is the same as $t < 5$ AND $t > -5$.

Graphing the solution set to an absolute value linear inequality gives you the same visual representation as you had when graphing the solution set to linear inequalities. The same rules apply when graphing absolute values of linear inequalities on a real number line. Once the solution is found, the open circle is used for absolute value inequalities containing the symbols >and <. The closed circle is used for absolute value inequalities containing the symbols $\leq$ and $\geq$.

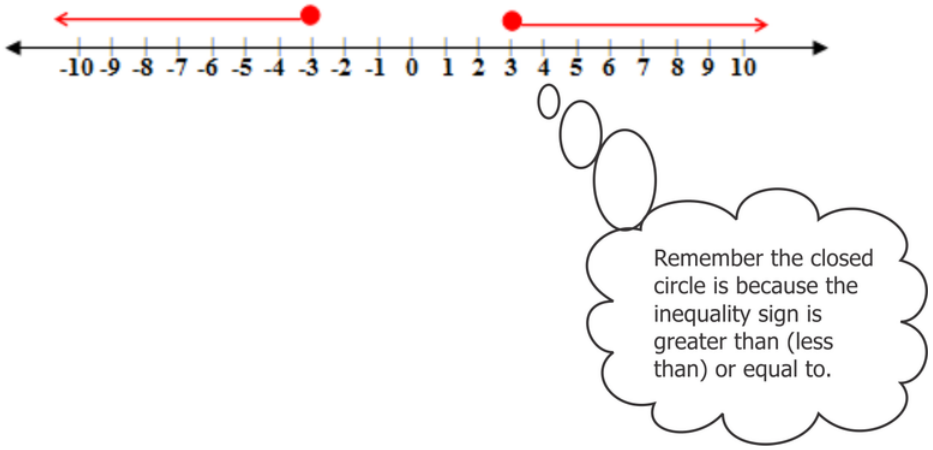
Example A

Represent the solution set to the following inequality on a number line: $|2x| \geq 6$.

Solution: First solve the inequality. Then, represent your solution on a number line.

$$
\begin{aligned}
|2x| &\geq 6 \\
2x &\geq 6 \\
\frac{2x}{2} &\geq \frac{6}{2} && \text{(Divide by 2 to isolate and solve for the variable)} \\
x &\geq 3 && \text{(Simplify)} \\
&OR \\
2x &\leq -6 \\
\frac{2x}{2} &\leq \frac{-6}{2} && \text{(Divide by 2 to isolate and solve for the variable)} \\
x &\leq -3 && \text{(Simplify)}
\end{aligned}
$$

The solution sets are $x \geq 3$ OR $x \leq -3$.



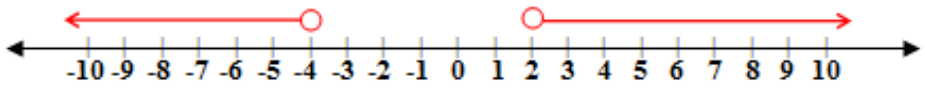
Example B

Solve the following inequality and graph the solution on a number line: $|x+1| > 3$

Solution: First solve the inequality. Then, represent your solution on a number line.

$$
\begin{aligned}
|x+1| &> 3 && \text{(Divide both sides by 2 to solve for the variable)} \\
x+1 &> 3 \\
x+1-1 &> 3-1 && \text{(Subtract 1 from both sides of the inequality sign)} \\
x &> 2 \\
&OR \\
x+1 &< -3 \\
x+1-1 &< -3-1 && \text{(Subtract 1 from both sides of the inequality sign)} \\
x &< -4
\end{aligned}
$$

The solution sets are $x > 2$, OR $x < -4$.



Example C

Solve the following inequality and graph the solution on a number line: $\left|x-\frac{5}{2}\right| < 1$

Solution: First solve the inequality. Then, represent your solution on a number line.

$$\left|x-\frac{5}{2}\right|<1$$

$$x-\frac{5}{2}<1$$

$$\left(\frac{2}{2}\right)x-\frac{5}{2}<\left(\frac{2}{2}\right)1$$

$$\frac{2x}{2}-\frac{5}{2}<\frac{2}{2}$$

$$2x-5<2 \quad \text{(Simplify)}$$

$$2x-5+5<2+5 \quad \text{(Add 5 to isolate the variable)}$$

$$2x<7 \quad \text{(Simplify)}$$

$$\frac{2x}{2}<\frac{7}{2}$$

$$x<\frac{7}{2}$$

OR

$$x-\frac{5}{2}>-1$$

$$\left(\frac{2}{2}\right)x-\frac{5}{2}>\left(\frac{2}{2}\right)(-1) \quad \text{(Multiply to get common denominator (LCD = 2))}$$

$$\frac{2x}{2}-\frac{5}{2}<\frac{-2}{2} \quad \text{(Simplify)}$$

$$2x-5>-2 \quad \text{(Simplify)}$$

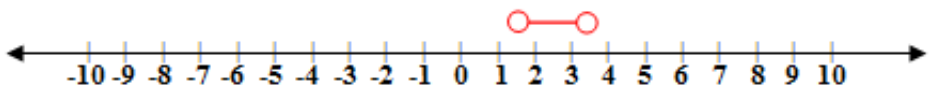
$$2x-5+5>-2+5 \quad \text{(Add 5 to isolate the variable)}$$

$$2x>3 \quad \text{(Simplify)}$$

$$\frac{2x}{2}>\frac{3}{2} \quad \text{(Divide both sides by 2 to solve for the variable)}$$

$$x>\frac{3}{2}$$

The solution is $\frac{3}{2}<x<\frac{7}{2}$.



Concept Problem Revisited

Solve the following inequality and graph the solution on a number line.

$|x+2|\leq 3$

First solve the inequality:

$$
\begin{aligned}
x+2 &\leq 3 \\
x+2-2 &\leq 3-2 && \text{Subtract 2 from both sides to isolate the variable} \\
x &\leq 1 && \text{Simplify} \\
&OR \\
x+2 &\geq -3 \\
x+2-2 &\geq -3-2 && \text{Subtract 2 from both sides to isolate the variable} \\
x &\geq -5 && \text{Simplify}
\end{aligned}
$$

The solution is $-5 \leq x \leq 1$.

Representing on a number line:

Guided Practice

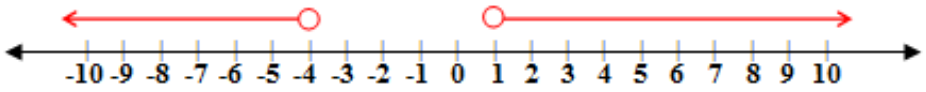
1. Represent the solution set to the inequality $|2x+3|>5$ on a number line.
2. Represent the solution set to the inequality $|32x-16| \geq 32$ on a number line.
3. Represent the solution set to the inequality $|x-21.5|>12.5$ on a number line.

Answers:

1. $|2x+3|>5$

$$
\begin{aligned}
2x+3 &> 5 \\
2x+3-3 &> 5-3 && \text{(Subtract 3 from both sides of the inequality sign)} \\
2x &> 2 && \text{(Simplify)} \\
\frac{2x}{2} &> \frac{2}{2} && \text{(Divide by 2 to solve for the variable)} \\
x &> 1 \\
&OR \\
2x+3 &< -5 \\
2x+3-3 &< -5-3 && \text{(Subtract 3 from both sides of the inequality sign)} \\
2x &< -8 && \text{(Simplify)} \\
\frac{2x}{2} &> \frac{-8}{2} && \text{(Divide by 2 to solve for the variable)} \\
x &< -4
\end{aligned}
$$

The solution sets are $x > 1$ or $x < -4$.



2. $|32x-16| \geq 32$

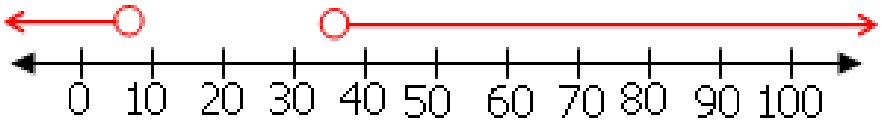
$$
\begin{aligned}
32x-16 &\geq 32 && \\
32x-16+16 &\geq 32+16 && \text{(Add 16 to both sides of the inequality sign)} \\
32x &\geq 48 && \text{(Simplify)} \\
\frac{32x}{32} &\geq \frac{48}{32} && \text{(Divide by 32 to solve for the variable)} \\
x &\geq \frac{3}{2} && \\
&OR && \\
32x-16 &\leq -32 && \\
32x-16+16 &\leq -32+16 && \text{(Add 16 to both sides of the inequality sign)} \\
32x &\leq -16 && \text{(Simplify)} \\
\frac{32x}{32} &\leq \frac{-16}{32} && \text{(Divide by 32 to solve for the variable)} \\
x &\leq -\frac{1}{2} &&
\end{aligned}
$$

The solution sets are $x \geq \frac{3}{2}$ or $x \leq -\frac{1}{2}$.

3. $|x-21.5| > 12.5$

$$
\begin{aligned}
x-21.5 &> 12.5 && \\
x-21.5+21.5 &> 12.5+21.5 && \text{(Add 21.5 to both sides to isolate the variable)} \\
x &> 34 && \text{(Simplify)} \\
&OR && \\
x-21.5 &< -12.5 && \\
x-21.5+21.5 &< -12.5+21.5 && \text{(Add 21.5 to both sides to isolate the variable)} \\
x &< 9 && \text{(Simplify)}
\end{aligned}
$$

The solution sets are $x < 9$ or $x > 34$.



Explore More

Represent the solution sets to each absolute value inequality on a number line.

1. $|3-2x|<3$
2. $2|\frac{2x}{3}+1|\geq 4$
3. $|\frac{2g-9}{4}|<1$
4. $|\frac{4}{3}x-5|\geq 7$
5. $|2x+5|+4\geq 7$
6. $|p-16|>10$
7. $|r+2|<5$
8. $|3-2k|\geq 1$
9. $|8-y|>5$
10. $8\geq |5d-2|$
11. $|s+2|-5>8$
12. $|10+8w|-2<16$
13. $|2q+1|-5\leq 7$
14. $|\frac{1}{3}(g-2)|<4$
15. $|-2(e+4)|>17$