Applications with Inequalities

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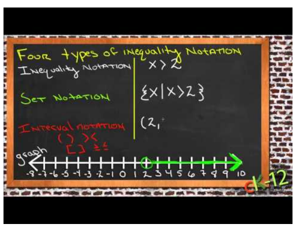
CHAPTER 1 Applications with Inequalities

Here you'll learn how to express the solutions of an inequality in four different ways. You'll also learn how to dentify the number of solutions an inequality has and you'll solve real-world problems using inequalities.

What if you solved an inequality and came up with the solution $x > -3$? How else could you express this solution? After completing this Concept, you'll be able to express the solution of an inequality in inequality notation, set notation, interval notation, and as a solution graph.

Watch This



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CK-12 Foundation: 0604S Using Inequalities (H264)

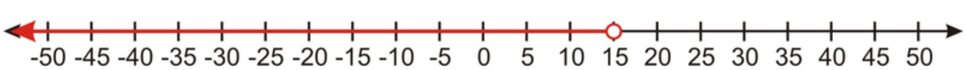
Guidance

Ms. Jerome wants to buy identical boxes of art supplies for her 25 students. If she can spend no more than \$375 on art supplies, what inequality describes the price can she afford for each individual box of supplies?

Expressing Solutions of an Inequality

The solution of an inequality can be expressed in four different ways:

1. **Inequality notation** The answer is simply expressed as $x < 15$.
2. **Set notation** The answer is expressed as a set: $\{x|x < 15\}$. The brackets indicate a set and the vertical line means "such that," so we read this expression as "the set of all values of x such that x is a real number less than 15".
3. **Interval notation** uses brackets to indicate the range of values in the solution. For example, the answer to our problem would be expressed as $(-\infty, 15)$, meaning "the interval containing all the numbers from $-\infty$ to 15 but not actually including $-\infty$ or 15".
   a. Square or **closed brackets** "[" and "]" indicate that the number next to the bracket is included in the solution set.
   b. Round or **open brackets** "(" and ")" indicate that the number next to the bracket is not included in the solution set. When using **infinity** and **negative infinity** (∞ and $-\infty$), we always use open brackets, because infinity isn't an actual number and so it can't ever really be included in an interval.
4. **Solution graph** shows the solution on the real number line. A closed circle on a number indicates that the number is included in the solution set, while an open circle indicates that the number is not included in the set. For our example, the solution graph is:



Example A

a) [-4, 6] means that the solution is all numbers between -4 and 6 **including** -4 and 6.

b) (8, 24) means that the solution is all numbers between 8 and 24 **not including** the numbers 8 and 24.

c) [3, 12) means that the solution is all numbers between 3 and 12, **including** 3 but **not including** 12.

d) $(-10, \infty)$ means that the solution is all numbers greater than -10, **not including** -10.

e) $(-\infty, \infty)$ means that the solution is all real numbers.

Identify the Number of Solutions of an Inequality

Inequalities can have:

- A set that has an infinite number of solutions.
- A set that has a discrete number of solutions.
- No solutions.

The inequalities we have solved so far all have an infinite number of solutions, at least in theory. For example, the inequality $\frac{5x-1}{4} > -2(x+5)$ has the solution $x > -3$. This solution says that all real numbers greater than -3 make this inequality true, and there are infinitely many such numbers.

However, in real life, sometimes we are trying to solve a problem that can only have positive integer answers, because the answers describe numbers of discrete objects.

For example, suppose you are trying to figure out how many $8 CDs you can buy if you want to spend less than $50. An inequality to describe this situation would be $8x < 50$, and if you solved that inequality you would get $x < \frac{50}{8}$, or $x < 6.25$.

But could you really buy *any* number of CDs as long as it's less than 6.25? No; you couldn't really buy 6.1 CDs, or -5 CDs, or any other fractional or negative number of CDs. So if we wanted to express our solution in set notation, we couldn't express it as the set of all numbers less than 6.25, or $\{x | x < 6.25\}$. Instead, the solution is just the set containing all the *nonnegative whole numbers* less than 6.25, or {0, 1, 2, 3, 4, 5, 6}. When we're solving a real-world problem dealing with discrete objects like CDs, our solution set will often be a finite set of numbers instead of an infinite interval.

An inequality can also have no solutions at all. For example, consider the inequality $x-5 > x+6$. When we subtract x from both sides, we end up with $-5 > 6$, which is not true for any value of x. We say that this inequality has no solution.

The opposite can also be true. If we flip the inequality sign in the above inequality, we get $x-5 < x+6$, which simplifies to $-5 < 6$. That's always true no matter what x is, so the solution to that inequality would be all real numbers, or $(-\infty, \infty)$.

Solve Real-World Problems Using Inequalities

Solving real-world problems that involve inequalities is very much like solving problems that involve equations.

Example B

In order to get a bonus this month, Leon must sell at least 120 newspaper subscriptions. He sold 85 subscriptions in the first three weeks of the month. How many subscriptions must Leon sell in the last week of the month?

Solution

Let $x =$ the number of subscriptions Leon sells in the last week of the month. The total number of subscriptions for the month must be greater than 120, so we write $85 + x \geq 120$. We solve the inequality by subtracting 85 from both sides: $x \geq 35$.

Leon must sell 35 or more subscriptions in the last week to get his bonus.

To check the answer, we see that $85+35=120$. If he sells 35 or more subscriptions, the total number of subscriptions he sells that month will be 120 or more. **The answer checks out**.

Example C

Virena's Scout troop is trying to raise at least $650 this spring. How many boxes of cookies must they sell at $4.50 per box in order to reach their goal?

Solution

Let $x =$ number of boxes sold. Then the inequality describing this problem is $4.50x \geq 650$.

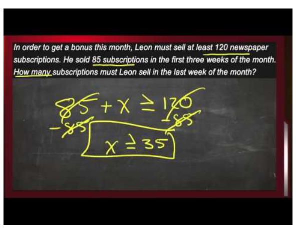
We solve the inequality by dividing both sides by 4.50: $x \geq 144.44$.

We round up the answer to 145 since only whole boxes can be sold.

Virena's troop must sell at least 145 boxes.

If we multiply 145 by $4.50 we obtain $652.50, so if Virena's troop sells more than 145 boxes they will raise more than $650. But if they sell 144 boxes, they will only raise $648, which is not enough. So they must indeed sell at least 145 boxes. **The answer checks out**.

Watch this video for help with the Examples above.



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CK-12 Foundation: Using Inequalities

Vocabulary

- Inequalities can have infinite solutions, no solutions, or discrete solutions.
- There are four ways to represent an inequality: *Equation notation, set notation, interval notation,* and *solution graph.*

Guided Practice

The width of a rectangle is 20 inches. What must the length be if the perimeter is at least 180 inches?

Solution

Let $x =$ length of the rectangle. The formula for perimeter is

$$\text{Perimeter} = 2 \times \text{length} + 2 \times \text{width}$$

Since the perimeter must be at least 180 inches, we have $2x+2(20) \geq 180$.

Simplify: $2x+40 \geq 180$

Subtract 40 from both sides: $2x \geq 140$

Divide both sides by 2: $x \geq 70$

The length must be at least 70 inches.

If the length is at least 70 inches and the width is 20 inches, then the perimeter is at least $2(70)+2(20) = 180$ *inches*. **The answer checks out**.

Explore More

Solve each inequality. Give the solution in inequality notation and interval notation.

1. $x+15 < 12$
2. $x-4 \geq 13$
3. $9x > -\frac{3}{4}$
4. $-\frac{x}{15} \leq 5$
5. $620x > 2400$
6. $\frac{x}{20} \geq -\frac{7}{40}$
7. $\frac{3x}{5} > \frac{3}{5}$
8. $x+3 > x-2$

Solve each inequality. Give the solution in inequality notation and set notation.

9. $x+17 < 3$
10. $x-12 \geq 80$
11. $-0.5x \leq 7.5$
12. $75x \geq 125$
13. $\frac{x}{-3} > -\frac{10}{9}$
14. $\frac{x}{-15} < 8$
15. $\frac{x}{4} > \frac{5}{4}$
16. $3x-7 \geq 3(x-7)$

Solve the following inequalities, give the solution in set notation, and show the solution graph.

17. $4x+3 < -1$
18. $2x < 7x-36$
19. $5x > 8x+27$
20. $5-x < 9+x$
21. $4-6x \leq 2(2x+3)$
22. $5(4x+3) \geq 9(x-2)-x$
23. $2(2x-1)+3 < 5(x+3)-2x$
24. $8x-5(4x+1) \geq -1+2(4x-3)$
25. 9. $2(7x-2)-3(x+2) < 4x-(3x+4)$
26. $\frac{2}{3}x-\frac{1}{2}(4x-1) \geq x+2(x-3)$
27. At the San Diego Zoo you can either pay $22.75 for the entrance fee or $71 for the yearly pass which entitles you to unlimited admission.
    a. At most how many times can you enter the zoo for the $22.75 entrance fee before spending more than the cost of a yearly membership?
    b. Are there infinitely many or finitely many solutions to this inequality?
28. Proteek's scores for four tests were 82, 95, 86, and 88. What will he have to score on his fifth and last test to average at least 90 for the term?