Write and Solve Multi-Step Inequalities Given Problem Situations

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CHAPTER 1 Write and Solve Multi-Step Inequalities Given Problem Situations

Here you'll learn to write and solve multi-step inequalities given problem situations.

The marching band at Floyd Middle School always performs during half-time of the football games. The band usually performs different routines that are an average of 6 minutes for each arrangement. Mrs. Kline likes the students to have a variety of different pieces to perform.

During the game, they always perform for at least 42 minutes but not more than 60 minutes. If the average time for each piece is 8 minutes, what is the least number of pieces that the band can perform? What is the greatest number of pieces that the band will perform?

This problem has a compound inequality in it. You will notice that there are two different inequalities being mentioned. This Concept will teach you how to translate verbal language into a compound inequality so that you can solve the inequality. Pay close attention, because you will see this problem again at the end of the Concept.

Guidance

Sometimes, however, to truly understand the values represented by a variable, we need consider two inequalities *together*.

Two inequalities considered together form a *compound inequality.*

Think about it this way, suppose we know that $n > 0$ *and* we also know that $n < 5$. In that case, we need to consider those two inequalities together. That means, we need to consider a compound inequality.

There are two types of compound inequalities you should know about.

- **A *conjunction* is a compound inequality that contains the word *and.*** A conjunction is true only if both inequalities are true.

$n > 0$ *and* $n < 5$

- **A *disjunction* is a compound inequality that contains the word *or.*** A disjunction is true if either of the inequalities is true.

We can write compound inequalities when we have words that describe more than one inequality. Let's look at one.

The sum of a number, n, and 4 is at least 12 and at most 20.

a. Translate the sentence above into a compound inequality.

b. Solve the compound inequality.

Consider part *a* first.

Break the sentence into parts and translate each part into an inequality.

$$\begin{array}{ccc} The\ \underline{sum\ of\ a\ number,\ n,\ and\ 4}\ is & \underline{at\ least} & \underline{12}\ldots \\ \downarrow & \downarrow & \downarrow \\ n+4 & \geq & 12 \end{array}$$

$$\begin{array}{ccc} The\ \underline{sum\ of\ a\ number,\ n,\ and\ 4}\ is \ldots & \underline{at\ most} & \underline{20}. \\ \downarrow & \downarrow & \downarrow \\ n+4 & \leq & 20 \end{array}$$

Look at the original sentence.

The sum of a number, *n*, and 4 is at least 12 <u>and</u> at most 20.

The word *and* which is underlined above shows that this sentence represents a conjunction.

So, the compound inequality could be written as: $n+4 \geq 12$ *and* $n+4 \leq 20$.

Or, we could rewrite $n+4 \geq 12$ as $12 \leq n+4$ and put the two inequalities together like this: $12 \leq n+4 \leq 20$.

Next, consider part *b* .

To solve the compound inequality, treat each inequality separately.

Subtract 4 from each side.

$$\begin{array}{cc} n+4 \geq 12 & n+4 \leq 20 \\ n+4-4 \geq 12-4 & n+4-4 \leq 20-4 \\ n+0 \geq 8 & n+0 \leq 16 \\ n \geq 8 & n \leq 16 \end{array}$$

So, the solution could be written as: $n \geq 8$ **and** $n \leq 16$**.**

It could also be written as: $8 \leq n \leq 16$**.**

Here is another real-world situation.

Brandon earns $7 per hour at his job. The number of hours he works varies from week to week. However, each week he always earns no less than $70 and no more than $140. Let h represent the number of hours he works each week.

a. Write a compound inequality to represent this problem situation.

b. Solve the inequality to determine the range in the number of hours Brandon works each week.

c. According to the problem, does Brandon ever work 25 hours in any given week? Explain.

Consider part *a* first.

Since Brandon earns $7 per hour, you can represent the number of dollars he earns each week by multiplying 7 by the number of hours he works. In other words, you can represent his total earnings as $7 \times h$ or $7h$.

Now, break the problem into parts and translate each part into an inequality.

$$\begin{array}{ccc} \ldots\underline{each\ week\ldots earns} & \underline{no\ less\ than} & \underline{\$70}\ldots \\ \downarrow & \downarrow & \downarrow \\ 7h & \geq & 70 \end{array}$$

$$\begin{array}{ccc} \ldots\underline{each\ week\ldots earns} & \underline{no\ more\ than} & \underline{\$140}. \\ \downarrow & \downarrow & \downarrow \\ 7h & \leq & 140 \end{array}$$

Look at the original sentence.

...each week he always earns no less than $70 and no more than $140.

Because of the word *and*, this compound inequality is an example of a conjunction.

We could represent this conjunction as: $7h \geq 70$ *and* $7h \leq 140$. We could also write this conjunction as: $70 \leq 7h \leq 140$.

Next, consider part *b* .

To solve the compound inequality, treat each inequality separately.

Divide each side by 7.

$$\begin{aligned} 7h &\geq 70 & 7h &\leq 140 \\ \frac{7h}{7} &\geq \frac{70}{7} & \frac{7h}{7} &\leq \frac{140}{7} \\ 1h &\geq 10 & 1h &\leq 20 \\ h &\geq 10 & h &\leq 20 \end{aligned}$$

So, the solution could be written as: $h \geq 10$ ***and*** $h \leq 20$**. It could also be written as:** $10 \leq h \leq 20$**.**

This solution shows that the number of hours Brandon works each week is greater than or equal to 10 *and* less than or equal to 20. In other words, Brandon works 10 to 20 hours in any given week.

Lastly, consider part *c* .

In part *b*, you determined that the number of hours Brandon works in any given week is always less than or equal to 20. This means he never works more than 20 hours in any given week. So, he would not work 25 hours during a particular week.

Example A

The sum of a number and 3 is at least 10 but not more than 25.

Solution: $10 \leq x+3 \leq 25$

Example B

The sum of a number and six is greater than four and less than 12.

Solution: $4 < x+6 < 12$

Example C

The product of a number times two is greater than fifteen and less than twenty.

Solution: $15 < 2x < 20$

Now let's go back to the dilemma from the beginning of the Concept.

First, let's write down the given information.

The average length of each piece is 6 minutes.

The band performs for no less than 42 minutes.

The band performs for no more than 60 minutes.

We need to figure out the range of the number of pieces performed. This is our variable p.

Here is the inequality.

$42 \le 6p \ge 60$

Now we can solve each inequality for the range. We will have a low range and a high range for our number of pieces.

$$\begin{aligned} 42 &\le 6p \\ 7 &\le p \\ 6p &\ge 60 \\ p &\ge 10 \end{aligned}$$

The band will perform between 7 and 10 pieces given the range of times that they are allowed to perform.

Guided Practice

Here is one for you to try on your own.

Six less than half of a number, *x*, is either less than -1 or greater than 10.

a. Translate the sentence above into a compound inequality.

b. Solve the compound inequality.

Solution

Consider part *a* first.

Break the sentence into parts and translate each part into an inequality.

$$\begin{array}{ccccc}
\underline{Six} & \underline{less\ than} & \underline{half\ of\ a\ number,\ x,} & is \ldots \underline{less\ than} & \underline{-1} \ldots \\
\searrow & \downarrow & \swarrow & \downarrow & \downarrow \\
\searrow & \downarrow & \swarrow & \downarrow & \downarrow \\
\searrow & \downarrow & \swarrow & \downarrow & \downarrow \\
\frac{x}{2} & - & 6 & < & -1
\end{array}$$

$$\begin{array}{ccccc}
\underline{Six} & \underline{less\ than} & \underline{half\ of\ a\ number,\ x,} & is \ldots \underline{greater\ than} & \underline{10}. \\
\searrow & \downarrow & \swarrow & \downarrow & \downarrow \\
\searrow & \downarrow & \swarrow & \downarrow & \downarrow \\
\searrow & \downarrow & \swarrow & \downarrow & \downarrow \\
\frac{x}{2} & - & 6 & > & 10
\end{array}$$

Look at the original sentence.

Six less than half of a number, x, is either greater than 10 or less than -1.

The word *or* which is underlined above shows that this sentence represents a disjunction.

So, the compound inequality could be written as: $\frac{x}{2} - 6 < -1$ *or* $\frac{x}{2} - 6 > 10$

Next, consider part *b* .

To solve the compound inequality, treat each inequality separately.

First, add 6 to each side.

$$\begin{aligned} \frac{x}{2} - 6 &< -1 \\ \frac{x}{2} - 6 + 6 &< -1 + 6 \\ \frac{x}{2} + (-6 + 6) &< 5 \\ \frac{x}{2} + 0 &< 5 \\ \frac{x}{2} &< 5 \end{aligned}$$

$$\begin{aligned} &\frac{x}{2} - 6 > 10 \\ &\frac{x}{2} - 6 + 6 > 10 + 6 \\ &\frac{x}{2} + (-6 + 6) > 16 \\ &\frac{x}{2} + 0 > 16 \\ &\frac{x}{2} > 16 \end{aligned}$$

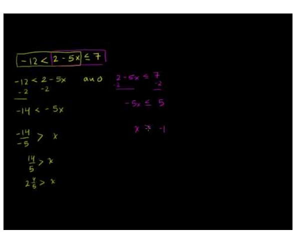
Next, multiply each side by 2.

$$\begin{aligned} \frac{x}{2} &< 5 \\ \frac{x}{2} \times 2 &< 5 \times 2 \\ \frac{x}{\cancel{2}} \times \frac{\cancel{2}}{1} &< 10 \\ \frac{x}{1} &< 10 \\ x &< 10 \end{aligned}$$

$$\begin{aligned} \frac{x}{2} &> 16 \\ \frac{x}{2} \times 2 &> 16 \times 2 \\ \frac{x}{\cancel{2}} \times \frac{\cancel{2}}{1} &> 32 \\ \frac{x}{1} &> 32 \\ x &> 32 \end{aligned}$$

So, the solution could be written as: $x < 10$ or $x > 32$.

Video Review



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Click image to the left or use the URL below.

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Khan Academy Compound Inequalities

Explore More

Directions: Use each example to work with compound inequalities.

Two less than a number, x, is at least 16 and at most 25

1. Translate the sentence into an inequality.
2. Solve the inequality.
3. Write a mathematical statement to show the range of possibilities for the answer.

One-third of a number, n, is either less than -5 or greater than 3.

4. Translate the sentence into an inequality.
5. Solve the inequality.
6. Write a mathematical statement to show the range of possibilities for the answer.

Seven more than twice a number, n, is either less than -5 or at least 9.

7. Translate the sentence into an inequality.
8. Solve the inequality.
9. Write a mathematical statement to show the range of possibilities for the answer.

Directions: Solve each problem.

When Harriet goes to the diner for lunch, she buys exactly two items: a wrap sandwich and a $3 milkshake. The total cost of her lunch is always more than $8 and less than $12. Let w represent the cost, in dollars, of any of the wrap sandwiches Harriet buys.

10. Write a compound inequality to represent this problem situation.
11. Solve the inequality you wrote in part a.
12. According to the problem, is it possible that Harriet sometimes buys a wrap sandwich that costs $10?
13. Why? Explain.

Mr. Jameson pays $3 per gallon for gasoline. Each week, the amount he spends on gas for his car is always at least $30 on gas and at most $105. Let g represent the number of gallons of gasoline he buys in any given week.

14. Write a compound inequality to represent this problem situation.
15. Solve the inequality you wrote in part a.