

PROPORTIONAL RELATIONSHIPS AND SLOPE

A **proportional relationship** between two quantities is one in which the two quantities vary directly with one and other. If one item is doubled, the other, related item is also doubled. Because of this, it is also called a direct variation. The equations of such relationships are always in the form $y = mx$, and when graphed produce a line that passes through the origin. In this equation, m is the **slope** of the line, and it is also called the unit rate, the rate of change, or the constant of proportionality of the function.

Examples

For each of the following relationships, graph the proportional relationship between the two quantities, write the equation representing the relationship, and describe how the unit rate, or slope is represented on the graph.

Example 1: Gasoline cost \$4.24 per gallon.

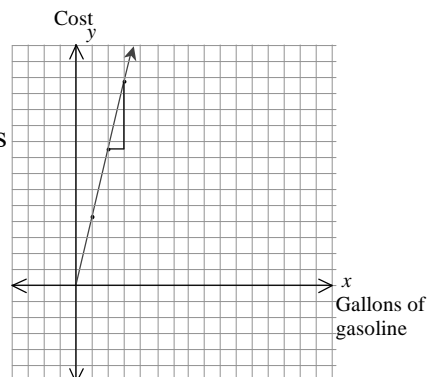
We can start by creating a table to show how these two quantities, gallons of gas and cost, vary.

Gallons of gas	0	1	2	3
Cost(\$)	0	4.24	8.48	12.72

Two things show us that this is definitely a proportional relationship. First, it contains the origin, $(0, 0)$, and this makes sense: if we buy zero gallons of gas it will cost zero dollars. Second, if the number of gallons is doubled, the cost is doubled; if it is tripled, the cost is tripled. The equation that will represent this data is $y = 4.24x$, where x is the number of gallons of gasoline and y is the total cost. The graph is shown below.

Note: The equation does not extend into the third quadrant because this region does not make sense for the situation. We will not buy negative quantities of gasoline, nor pay for it with negative dollars!

There are different ways to determine the slope of this line. First, we reasoned what the slope was when we determined the equation of the line: for each gallon of gas the cost increases \$4.24. Also, we can find the slope by creating a “slope triangle” which represents $\frac{\text{rise}}{\text{run}}$. In the slope triangle drawn at right, the “rise,” or vertical change, is 4.24 while the “run,” or horizontal change, is 1. Therefore the slope is $\frac{4.24}{1} = 4.24$. Either way, the constant of proportionality is the slope, which is 4.24.



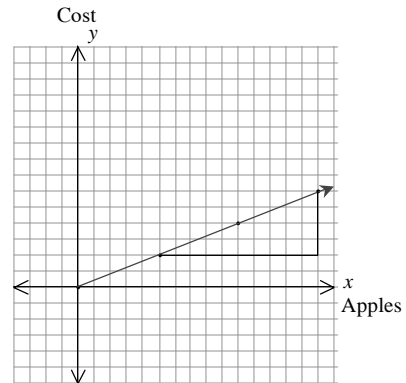
Example 2: Five Gala apples cost \$2.00.

Again, we can begin by creating a table relating the number of apples to their cost.

# of Apples	0	5	10	15
Cost(\$)	0	2.00	4.00	6.00

Just as with the last example, this contains the origin, and when the number of apples is doubled so is the cost. In this example, it is not quite as obvious what the equation and slope are. Therefore, let's plot these points and see if we can determine the slope that way.

Using the slope triangle, we see that the slope is $\frac{\text{rise}}{\text{run}} = \frac{4}{10} = \frac{2}{5}$. Therefore the equation of this relationship is $y = \frac{2}{5}x$. The slope is also the unit rate or constant rate of change: for every five apples, we pay another \$2.00. To see this as a unit rate, we need to know the cost of one apple. If five apples cost \$2.00, then one apple costs $\frac{2.00}{5} = 0.40$, or \$0.40 for each apple. This is also represented on the graph: for each apple, the graph rises only 0.40.

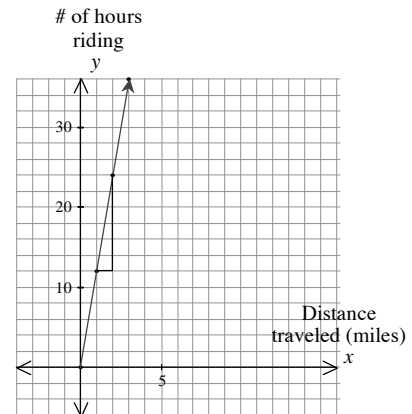


Example 3: Tess rides her bike at 12 mph.

Let's start with a table.

# of hours riding	0	1	2	3
Distance traveled (miles)	0	12	24	36

We are relating the number of hours Tess rides her bike to the distance she has traveled. This is a proportional relationship: it passes through the origin, and if the number of hours is doubled or tripled, the distance Tess travels is also doubled or tripled. Here, the equation representing this data is $y = 12x$ where x is the number of hours of riding, and y is the number of miles she has traveled. The graph at right contains a slope triangle. The slope, or unit change, is $\frac{\text{rise}}{\text{run}} = \frac{12}{1}$ or simply 12.



Problems

For each of the following problems, draw the graph of the proportional relationship between the two quantities and describe how the unit rate is represented on the graph.

1. An Elm tree grows 8 inches each year.
2. Davis adds \$3.00 to his savings account each week.
3. Bananas are \$2.40 per pound.
4. Lunches in the cafeteria are \$2.25 each.
5. The Dry Cleaners charges \$13.00 to clean and press two jackets.
6. Professor McGonnagal grades six exams every hour.
7. Bobby Pendragon travels 23 miles each day across Denduron.
8. Amelia Bedelia makes three pies every hour.
9. Every six days, Draco receives four boxes of cauldron cakes.
10. For every \$10 Tess adds to her savings, her parents add \$1.50.
11. Sabrina Grimm eats 2 cups of purple mashed potatoes every 20 minutes.
12. Milo drives through four tollbooths every 30 minutes.

Answers

1. The graph of $y = 8x$, which is a line passing through $(0, 0)$ with a slope of 8; the slope 8 is the rate of change of the tree each year.
2. The graph of $y = 3x$, which is a line passing through $(0, 0)$ with a slope of 3; the slope 3 is the rate of change of Davis' account each week.
3. The graph of $y = 2.4x$, which is a line passing through $(0, 0)$ with a slope of 2.4; the slope 2.4 is the unit rate of each pound of bananas.
4. The graph of $y = 2.25x$, which is a line passing through $(0, 0)$ with a slope of 2.25; the slope 2.25 is the unit rate of change of each lunch.
5. The graph of $y = 6.5x$, which is a line passing through $(0, 0)$ with a slope of 6.5; the slope 6.5 is the unit rate of each jacket.
6. The graph of $y = 6x$, which is a line passing through $(0, 0)$ with a slope of 6; the slope 6 is the number of exams she grades each hour.
7. The graph of $y = 23x$, which is a line passing through $(0, 0)$ with a slope of 23; the slope 23 is the rate Bobby travels each day.
8. The graph of $y = 3x$, which is a line passing through $(0, 0)$ with a slope of 3; the slope 3 is the number of pies she makes each hour.
9. The graph of $y = \frac{4}{6}x = \frac{2}{3}x$, which is a line passing through $(0, 0)$ with a slope of $\frac{2}{3}$; the slope $\frac{2}{3}$ is the unit rate that Draco receives boxes.
10. The graph of $y = \frac{1.50}{10}x = 0.15x$, which is a line passing through $(0, 0)$ with a slope of 0.15; the slope 0.15 is the unit rate that Tess' parents add to her savings.
11. The graph of $y = \frac{2}{20}x = \frac{1}{10}x$, which is a line passing through $(0, 0)$ with a slope of $\frac{1}{10}$; the slope $\frac{1}{10}$ is the unit rate that Sabrina eats purple mashed potatoes.
12. The graph of $y = \frac{4}{30}x = \frac{2}{15}x$, which is a line passing through $(0, 0)$ with a slope of $\frac{2}{15}$; the slope $\frac{2}{15}$ is the unit rate that Milo drives through tollbooths.