Ratios and Rates: Ratios and Rates*

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Abstract

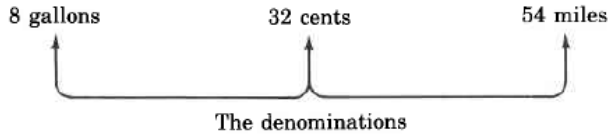
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses ratios and rates. By the end of the module students should be able to distinguish between denominate and pure numbers and between ratios and rates.

1 Section Overview

- Denominate Numbers and Pure Numbers
- Ratios and Rates

2 Denominate Numbers and Pure Numbers

Denominate Numbers, Like and Unlike Denominate Numbers
It is often necessary or convenient to compare two quantities**. Denominate numbers** are numbers together with some specified unit. If the units being compared are alike, the denominate numbers are called **like denominate numbers**. If units are not alike, the numbers are called **unlike denominate numbers**. Examples of denominate numbers are shown in the diagram:



Pure Numbers
Numbers that exist purely as numbers and do *not* represent amounts of quantities are called **pure numbers**. Examples of pure numbers are 8, 254, 0, $21\frac{5}{8}$, $\frac{2}{5}$, and 0.07.

Numbers can be *compared* in two ways: subtraction and division.

Comparing Numbers by Subtraction and Division
Comparison of two numbers by subtraction indicates how *much more* one number is than another.
Comparison by division indicates how *many times* larger or smaller one number is than another.
Comparing Pure or Like Denominate Numbers by Subtraction
Numbers can be compared by subtraction if and only if they both are like denominate numbers or both pure numbers.

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2.1 Sample Set A

Example 1

Compare 8 miles and 3 miles by subtraction.

8 mile − 3 miles = 5 miles

This means that 8 miles is 5 miles more than 3 miles.

Examples of use: I can now jog 8 miles whereas I used to jog only 3 miles. So, I can now jog 5 miles more than I used to.

Example 2

Compare 12 and 5 by subtraction.

$12 - 5 = 7$

This means that 12 is 7 more than 5.

Example 3

Comparing 8 miles and 5 gallons by subtraction makes no sense.

8 miles − 5 gallons =?

Example 4

Compare 36 and 4 by division.

$36 \div 4 = 9$

This means that 36 is 9 times as large as 4. Recall that $36 \div 4 = 9$ can be expressed as $\frac{36}{4} = 9$.

Example 5

Compare 8 miles and 2 miles by division.

$\frac{8 \text{ miles}}{2 \text{ miles}} = 4$

This means that 8 miles is 4 times as large as 2 miles.

Example of use: I can jog 8 miles to your 2 miles. Or, for every 2 miles that you jog, I jog 8. So, I jog 4 times as many miles as you jog.

Notice that when like quantities are being compared by division, we drop the units. Another way of looking at this is that the units divide out (cancel).

Example 6

Compare 30 miles and 2 gallons by division.

$\frac{30 \text{ miles}}{2 \text{ gallons}} = \frac{15 \text{ miles}}{1 \text{ gallon}}$

Example of use: A particular car goes 30 miles on 2 gallons of gasoline. This is the same as getting 15 miles to 1 gallon of gasoline.

Notice that when the quantities being compared by division are unlike quantities, we do not drop the units.

2.2 Practice Set A

Make the following comparisons and interpret each one.

Exercise 1 ***(Solution on p. 7.)***

Compare 10 diskettes to 2 diskettes by

(a) subtraction:
(b) division:

Exercise 2 ***(Solution on p. 7.)***

Compare, if possible, 16 bananas and 2 bags by

(a) subtraction:
(b) division:

3 Ratios and Rates

Ratio

A comparison, by division, of two pure numbers or two like denominate numbers is a **ratio**.

The comparison by division of the pure numbers $\frac{36}{4}$ and the like denominate numbers $\frac{8 \text{ miles}}{2 \text{ miles}}$ are examples of ratios.

Rate

A comparison, by division, of two unlike denominate numbers is a **rate**.

The comparison by division of two unlike denominate numbers, such as
$\frac{55 \text{ miles}}{1 \text{ gallon}}$ and $\frac{40 \text{ dollars}}{5 \text{ tickets}}$
are examples of rates.

Let's agree to represent two numbers (pure or denominate) with the letters a and b. This means that we're letting a represent some number and b represent some, perhaps different, number. With this agreement, we can write the ratio of the two numbers a and b as

$\frac{a}{b}$ or $\frac{b}{a}$

The ratio $\frac{a}{b}$ is read as " a to b."

The ratio $\frac{b}{a}$ is read as " b to a."

Since a ratio or a rate can be expressed as a fraction, it may be reducible.

3.1 Sample Set B

Example 7

The ratio 30 to 2 can be expressed as $\frac{30}{2}$. Reducing, we get $\frac{15}{1}$.

The ratio 30 to 2 is *equivalent* to the ratio 15 to 1.

Example 8

The rate "4 televisions to 12 people" can be expressed as $\frac{4 \text{ televisions}}{12 \text{ people}}$. The meaning of this rate is that "for every 4 televisions, there are 12 people."

Reducing, we get $\frac{1 \text{ television}}{3 \text{ people}}$. The meaning of this rate is that "for every 1 television, there are 3 people."

Thus, the rate of "4 televisions to 12 people" is the *same* as the rate of "1 television to 3 people."

3.2 Practice Set B

Write the following ratios and rates as fractions.

Exercise 3 *(Solution on p. 7.)*

3 to 2

Exercise 4 *(Solution on p. 7.)*

1 to 9

Exercise 5 *(Solution on p. 7.)*

5 books to 4 people

Exercise 6 *(Solution on p. 7.)*

120 miles to 2 hours

Exercise 7 *(Solution on p. 7.)*

8 liters to 3 liters

Write the following ratios and rates in the form "a to b." Reduce when necessary.

Exercise 8 *(Solution on p. 7.)*
$\frac{9}{5}$

Exercise 9 *(Solution on p. 7.)*
$\frac{1}{3}$

Exercise 10 *(Solution on p. 7.)*
$\frac{25 \text{ miles}}{2 \text{ gallons}}$

Exercise 11 *(Solution on p. 7.)*
$\frac{2 \text{ mechanics}}{4 \text{ wrenches}}$

Exercise 12 *(Solution on p. 7.)*
$\frac{15 \text{ video tapes}}{18 \text{ video tapes}}$

4 Exercises

For the following 9 problems, complete the statements.

Exercise 13 *(Solution on p. 7.)*
Two numbers can be compared by subtraction if and only if _____.

Exercise 14
A comparison, by division, of two pure numbers or two like denominate numbers is called a _____.

Exercise 15 *(Solution on p. 7.)*
A comparison, by division, of two unlike denominate numbers is called a _____.

Exercise 16
$\frac{6}{11}$ is an example of a _____. (ratio/rate)

Exercise 17 *(Solution on p. 7.)*
$\frac{5}{12}$ is an example of a _____. (ratio/rate)

Exercise 18
$\frac{7 \text{ erasers}}{12 \text{ pencils}}$ is an example of a _____. (ratio/rate)

Exercise 19 *(Solution on p. 7.)*
$\frac{20 \text{ silver coins}}{35 \text{ gold coins}}$ is an example of a _____.(ratio/rate)

Exercise 20
$\frac{3 \text{ sprinklers}}{5 \text{ sprinklers}}$ is an example of a _____. (ratio/rate)

Exercise 21 *(Solution on p. 7.)*
$\frac{18 \text{ exhaust valves}}{11 \text{ exhaust valves}}$ is an example of a _____.(ratio/rate)

For the following 7 problems, write each ratio or rate as a verbal phrase.

Exercise 22
$\frac{8}{3}$

Exercise 23 *(Solution on p. 7.)*
$\frac{2}{5}$

Exercise 24
$\frac{8 \text{ feet}}{3 \text{ seconds}}$

Exercise 25 *(Solution on p. 7.)*
$\frac{29 \text{ miles}}{2 \text{ gallons}}$

Exercise 26
$\frac{30{,}000 \text{ stars}}{300 \text{ stars}}$

Exercise 27 *(Solution on p. 7.)*
$\frac{5 \text{ yards}}{2 \text{ yards}}$

Exercise 28
$\frac{164 \text{ trees}}{28 \text{ trees}}$

For the following problems, write the simplified fractional form of each ratio or rate.

Exercise 29 *(Solution on p. 7.)*
12 to 5

Exercise 30
81 to 19

Exercise 31 *(Solution on p. 7.)*
42 plants to 5 homes

Exercise 32
8 books to 7 desks

Exercise 33 *(Solution on p. 7.)*
16 pints to 1 quart

Exercise 34
4 quarts to 1 gallon

Exercise 35 *(Solution on p. 8.)*
2.54 cm to 1 in

Exercise 36
80 tables to 18 tables

Exercise 37 *(Solution on p. 8.)*
25 cars to 10 cars

Exercise 38
37 wins to 16 losses

Exercise 39 *(Solution on p. 8.)*
105 hits to 315 at bats

Exercise 40
510 miles to 22 gallons

Exercise 41 *(Solution on p. 8.)*
1,042 characters to 1 page

Exercise 42
1,245 pages to 2 books

4.1 Exercises for Review

Exercise 43 *(Solution on p. 8.)*
(here[1]) Convert $\frac{16}{3}$ to a mixed number.

Exercise 44
(here[2]) $1\frac{5}{9}$ of $2\frac{4}{7}$ is what number?

[1] "Introduction to Fractions and Multiplication and Division of Fractions: Proper Fractions, Improper Fractions, and Mixed Numbers" <http://cnx.org/content/m34912/latest/>

[2] "Introduction to Fractions and Multiplication and Division of Fractions: Applications Involving Fractions" <http://cnx.org/content/m34930/latest/>

Exercise 45 *(Solution on p. 8.)*

(here[3]) Find the difference. $\frac{11}{28} - \frac{7}{45}$.

Exercise 46

(here[4]) Perform the division. If no repeating patterns seems to exist, round the quotient to three decimal places: $22.35 \div 17$

Exercise 47 *(Solution on p. 8.)*

(here[5]) Find the value of $1.85 + \frac{3}{8} \cdot 4.1$

[3] "Addition and Subtraction of Fractions, Comparing Fractions, and Complex Fractions: Addition and Subtraction of Fractions with Unlike Denominators" <http://cnx.org/content/m34935/latest/>

[4] "Decimals: Nonterminating Divisions" <http://cnx.org/content/m34969/latest/>

[5] "Decimals: Combinations of Operations with Decimals and Fractions" <http://cnx.org/content/m34971/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 2)

(a) 8 diskettes; 10 diskettes is 8 diskettes more than 2 diskettes.
(b) 5; 10 diskettes is 5 times as many diskettes as 2 diskettes.

Solution to Exercise (p. 2)

(a) Comparison by subtraction makes no sense.
(b) $\frac{16 \text{ bananas}}{2 \text{ bags}} = \frac{8 \text{ bananas}}{\text{bag}}$, 8 bananas per bag.

Solution to Exercise (p. 3)
$\frac{3}{2}$
Solution to Exercise (p. 3)
$\frac{1}{9}$
Solution to Exercise (p. 3)
$\frac{5 \text{ books}}{4 \text{ people}}$
Solution to Exercise (p. 3)
$\frac{60 \text{ miles}}{1 \text{ hour}}$
Solution to Exercise (p. 3)
$\frac{8}{3}$
Solution to Exercise (p. 3)
9 to 5
Solution to Exercise (p. 4)
1 to 3
Solution to Exercise (p. 4)
25 miles to 2 gallons
Solution to Exercise (p. 4)
1 mechanic to 2 wrenches
Solution to Exercise (p. 4)
5 to 6
Solution to Exercise (p. 4)
They are pure numbers or like denominate numbers.
Solution to Exercise (p. 4)
rate
Solution to Exercise (p. 4)
ratio
Solution to Exercise (p. 4)
rate
Solution to Exercise (p. 4)
ratio
Solution to Exercise (p. 4)
two to five
Solution to Exercise (p. 4)
29 mile per 2 gallons or $14\frac{1}{2}$ miles per 1 gallon
Solution to Exercise (p. 4)
5 to 2
Solution to Exercise (p. 5)
$\frac{12}{5}$
Solution to Exercise (p. 5)
$\frac{42 \text{ plants}}{5 \text{ homes}}$

Solution to Exercise (p. 5)

$\frac{16 \text{ pints}}{1 \text{ quart}}$

Solution to Exercise (p. 5)

$\frac{2.54 \text{ cm}}{1 \text{ inch}}$

Solution to Exercise (p. 5)

$\frac{5}{2}$

Solution to Exercise (p. 5)

$\frac{1 \text{ hit}}{3 \text{ at bats}}$

Solution to Exercise (p. 5)

$\frac{1{,}042 \text{ characters}}{1 \text{ page}}$

Solution to Exercise (p. 5)

$5\frac{1}{3}$

Solution to Exercise (p. 6)

$\frac{299}{1260}$

Solution to Exercise (p. 6)

3.3875