# The Distributive Property

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**AUTHORS**
CK12 Editor

CONCEPT 1

# The Distributive Property

## Learning Objectives

- Apply the distributive property.
- Identify parts of an expression.
- Solve real-world problems using the distributive property.

## Introduction

At the end of the school year, an elementary school teacher makes a little gift bag for each of his students. Each bag contains one class photograph, two party favors and five pieces of candy. The teacher will distribute the bags among his 28 students. How many of each item does the teacher need?

## Apply the Distributive Property

When we have a problem like the one posed in the introduction, **The Distributive Property** can help us solve it. First, we can write an expression for the contents of each bag: Items = (photo + 2 favors + 5 candies), or simply $I = (p + 2f + 5c)$.

For all 28 students, the teacher will need 28 times that number of items, so $I = 28(p + 2f + 5c)$.

Next, **the Distributive Property of Multiplication** tells us that when we have a single term multiplied by a sum of several terms, we can rewrite it by multiplying the single term by each of the other terms separately. In other words, $28(p + 2f + 5c) = 28(p) + 28(2f) + 28(5c)$, which simplifies to $28p + 56f + 140c$. So the teacher needs 28 class photos, 56 party favors and 140 pieces of candy.

You can see why the Distributive Property works by looking at a simple problem where we just have numbers inside the parentheses, and considering the **Order of Operations**.

**Example 1**

*Determine the value of 11(2 - 6) using both the Order of Operations and the Distributive Property.*

**Solution**

Order of Operations tells us to evaluate the amount inside the parentheses first:

$$11(2-6) = 11(-4) = -44$$

Now let's try it with the Distributive Property:

$$11(2-6) = 11(2) - 11(6) = 22 - 66 = -44$$

**Note:** When applying the Distributive Property you **MUST** take note of any **negative signs!**

**Example 2**

*Use the Distributive Property to determine the following.*

a) $11(2x+6)$

b) $7(3x-5)$

c) $\frac{2}{7}(3y^2-11)$

d) $\frac{2x}{7}\left(3y^2-\frac{11}{xy}\right)$

**Solution**

a) $11(2x+6)=11(2x)+11(6)=22x+66$

b) Note the negative sign on the second term.

$$7(3x-5)=21x-35$$

c) $\frac{2}{7}(3y^2-11)=\frac{2}{7}(3y^2)+\frac{2}{7}(-11)=\frac{6y^2}{7}-\frac{22}{7}$, or $\frac{6y^2-22}{7}$

d) $\frac{2x}{7}\left(3y^2-\frac{11}{xy}\right)=\frac{2x}{7}(3y^2)+\frac{2x}{7}\left(-\frac{11}{xy}\right)=\frac{6xy^2}{7}-\frac{22x}{7xy}$

We can simplify this answer by canceling the $x$'s in the second fraction, so we end up with $\frac{6xy^2}{7}-\frac{22}{7y}$.

## Identify Expressions That Involve the Distributive Property

The Distributive Property can also appear in expressions that don't include parentheses. In Lesson 1.2, we saw how the fraction bar also acts as a grouping symbol. Now we'll see how to use the Distributive Property with fractions.

**Example 3**

*Simplify the following expressions.*

a) $\frac{2x+8}{4}$

b) $\frac{9y-2}{3}$

c) $\frac{z+6}{2}$

**Solution**

Even though these expressions aren't written in a form we usually associate with the Distributive Property, remember that we treat the numerator of a fraction as if it were in parentheses, and that means we can use the Distributive Property here too.

a) $\frac{2x+8}{4}$ can be re-written as $\frac{1}{4}(2x+8)$. Then we can distribute the $\frac{1}{4}$:

$$\frac{1}{4}(2x+8)=\frac{2x}{4}+\frac{8}{4}=\frac{x}{2}+2$$

b) $\frac{9y-2}{3}$ can be re-written as $\frac{1}{3}(9y-2)$, and then we can distribute the $\frac{1}{3}$:

$$\frac{1}{3}(9y-2) = \frac{9y}{3} - \frac{2}{3} = 3y - \frac{2}{3}$$

c) Rewrite $\frac{z+6}{2}$ as $\frac{1}{2}(z+6)$, and distribute the $\frac{1}{2}$:

$$\frac{1}{2}(z+6) = \frac{z}{2} + \frac{6}{2} = \frac{z}{2} + 3$$

## Solve Real-World Problems Using the Distributive Property

The Distributive Property is one of the most common mathematical properties used in everyday life. Any time we have two or more groups of objects, the Distributive Property can help us solve for an unknown.

**Example 4**

*Each student on a field trip into a forest is to be given an emergency survival kit. The kit is to contain a flashlight, a first aid kit, and emergency food rations. Flashlights cost $12 each, first aid kits are $7 each and emergency food rations cost $2 per day. There is $500 available for the kits and 17 students to provide for. How many days worth of rations can be provided with each kit?*

The unknown quantity in this problem is the number of days' rations. This will be $x$ in our expression.

Each kit will contain **one** $12 flashlight, **one** $7 first aid kit, and $x$ times $2 worth of rations, for a total cost of $(12+7+2x)$ dollars. With 17 kits, therefore, the total cost will be $17(12+7+2x)$ dollars.

We can use the Distributive Property on this expression:

$$17(12+7+2x) = 204+119+34x$$

Since the total cost can be at most $500, we set the expression equal to 500 and solve for $x$. (You'll learn in more detail how to solve equations like this in the next chapter.)

$$\begin{aligned} 204+119+34x &= 500 \\ 323+34x &= 500 \\ 323+34x-323 &= 500-323 \\ 34x &= 177 \\ \frac{34x}{34} &= \frac{177}{34} \\ x &\approx 5.206 \end{aligned}$$

Since this represents the number of days' worth of rations that can be bought, we must **round to the next lowest whole number**. We wouldn't have enough money to buy a sixth day of supplies.

**Solution**

Five days worth of emergency rations can be purchased for each survival kit.

## Lesson Summary

- **Distributive Property** The product of a number and the sum of two numbers is equal to the first number times the second number plus the first number times the third number.
- When applying the Distributive Property you **MUST** take note of any **negative signs!**

## Further Practice

For more practice using the Distributive Property, try playing the Battleship game at http://www.quia.com/ba/15357.html.

## Review Questions

1. Use the Distributive Property to simplify the following expressions.
   a. $(x+4)-2(x+5)$
   b. $\frac{1}{2}(4z+6)$
   c. $(4+5)-(5+2)$
   d. $x(x+7)$
   e. $y(x+7)$
   f. $13x(3y+z)$
   g. $x\left(\frac{3}{x}+5\right)$
   h. $xy\left(\frac{1}{x}+\frac{2}{y}\right)$
2. Use the Distributive Property to remove the parentheses from the following expressions.
   a. $\frac{1}{2}(x-y)-4$
   b. $0.6(0.2x+0.7)$
   c. $6+(x-5)+7$
   d. $6-(x-5)+7$
   e. $4(m+7)-6(4-m)$
   f. $-5(y-11)+2y$
   g. $-(x-3y)+\frac{1}{2}(z+4)$
   h. $\frac{a}{b}\left(\frac{2}{a}+\frac{3}{b}+\frac{b}{5}\right)$
3. Use the Distributive Property to simplify the following fractions.
   a. $\frac{8x+12}{4}$
   b. $\frac{9x+12}{3}$
   c. $\frac{11x+12}{2}$
   d. $\frac{3y+2}{6}$
   e. $-\frac{6z-2}{3}$
   f. $\frac{7-6p}{3}$
   g. $\frac{3d-4}{6d}$
   h. $\frac{12g+8h}{4gh}$
4. A bookcase has five shelves, and each shelf contains seven poetry books and eleven novels. How many of each type of book does the bookcase contain?

5. Amar is making giant holiday cookies for his friends at school. He makes each cookie with 6 oz of cookie dough and decorates them with macadamia nuts. If Amar has 5 lbs of cookie dough $(1\ lb = 16\ oz)$ and 60 macadamia nuts, calculate the following.
   a. How many (full) cookies he can make?
   b. How many macadamia nuts he can put on each cookie, if each is to be identical?
   c. If 4 cups of flour and 1 cup of sugar went into each pound of cookie dough, how much of each did Amar use to make the 5 pounds of dough?