Function Notation

Andrew Gloag
Anne Gloag
Melissa Kramer

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Printed: July 24, 2012

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AUTHORS
Andrew Gloag
Anne Gloag
Melissa Kramer

EDITORS
Annamaria Farbizio

CONCEPT 1

Function Notation

Here you'll learn how to take an equation or inequality and rewrite it as a function. You'll also find out the difference between an independent variable and a dependent variable.

Suppose that you want to set up a function that allows you to input a dog's age in human years and which outputs the dog's age in dog years. How would you go about setting up such a function, and what notation would you use? Would the notation be the same as it was for the equations that you've looked at in previous Concepts? In this Concept, you'll learn what is needed to write a function such as this.

Guidance

Instead of purchasing a one-day ticket to the theme park, Joseph decided to pay by ride. Each ride costs $2.00. To describe the amount of money Joseph will spend, several mathematical concepts can be used.

First, an expression can be written to describe the relationship between the cost per ride and the number of rides, r. An equation can also be written if the total amount he wants to spend is known. An inequality can be used if Joseph wanted to spend less than a certain amount.

Example A

Using Joseph's situation, write the following:

a. An expression representing his total amount spent

b. An equation representing his total amount spent

c. An equation that shows Joseph wants to spend exactly $22.00 on rides

d. An inequality that describes the fact that Joseph will not spend more than $26.00 on rides

Solution: The variable in this situation is the number of rides Joseph will pay for. Call this r.

a. $2(r)$

b. $2(r) = m$

c. $2(r) = 22$

d. $2(r) \leq 26$

In addition to an expression, equation, or inequality, Joseph's situation can be expressed in the form of a function or a table.

Definition: A **function** is a relationship between two variables such that the input value has ONLY one output value.

Writing Equations as Functions

A function is a set of ordered pairs in which the first coordinate, usually x, matches with exactly one second coordinate, y. Equations that follow this definition can be written in function notation. The y coordinate represents the **dependent variable**, meaning the values of this variable depend upon what is substituted for the other variable.

Consider Joseph's equation $m = 2r$. Using function notation, the value of the equation (the money spent m) is replaced with $f(r)$. f represents the function name and (r) represents the variable. In this case the parentheses do not mean multiplication; rather, they separate the function name from the **independent variable.**

$$\begin{array}{c} input \\ \downarrow \\ \underbrace{f(x)}_{function\ box} = y \leftarrow output \end{array}$$

Example B

Rewrite the following equations in function notation.

a. $y = 7x - 3$

b. $d = 65t$

c. $F = 1.8C + 32$

Solution:

a. According to the definition of a function, $y = f(x)$, so $f(x) = 7x - 3$.

b. This time the dependent variable is d. Function notation replaces the dependent variable, so $d = f(t) = 65t$.

c. $F = f(C) = 1.8C + 32$

Why Use Function Notation?

Why is it necessary to use function notation? The necessity stems from using multiple equations. Function notation allows one to easily decipher between the equations. Suppose Joseph, Lacy, Kevin, and Alfred all went to the theme park together and chose to pay $2.00 for each ride. Each person would have the same equation $m = 2r$. Without asking each friend, we could not tell which equation belonged to whom. By substituting function notation for the dependent variable, it is easy to tell which function belongs to whom. By using function notation, it will be much easier to graph multiple lines.

Example C

Write functions to represent the total each friend spent at the park.

Solution:

$J(r) = 2r$ represents Joseph's total,

$L(r) = 2r$ represents Lacy's total,

$K(r) = 2r$ represents Kevin's total, and

$A(r) = 2r$ represents Alfred's total.

Vocabulary

Function: A ***function*** is a relationship between two variables such that the input value has ONLY one output value.

Dependent variable: A ***dependent variable*** is one whose values depend upon what is substituted for the other variable.

Independent variable: The ***independent variable*** is the variable which is not dependent on another variable. The dependent variable is dependent on the independent variable.

Guided Practice

Recall the example from a previous Concept where a student organization sells shirts to raise money. The cost of printing the shirts was expressed as $100 + 7x$ and for the revenue, we had the expression $15x$, where x is the number of shirts.

a. Write two functions, one for the cost and one for revenue.

b. Express that the cost must be less than or equal to $800.

c. Express that the revenue must be equal to $1500.

d. How many shirts must the students sell in order to make $1500?

Solution:

a. The cost function we will write as $C(x) = 100 + 7x$ and the revenue function we will write as $R(x) = 15x$.

b. Since $C(x)$ represents the costs, we substitute in $800 for $C(x)$ and replace the equation with the appropriate inequality symbol

$$100 + 7x \leq 800$$

This reads that $100 + 7x$ is less than or equal to $800, so we have written the inequality correctly.

c. We substitute in $1500 for $R(x)$, getting

$1500 = 15x$.

d. We want to find the value of x that will make this equation true. It looks like 100 is the answer. Checking this we see that 100 does satisfy the equation. The students must sell 100 shirts in order to have a revenue of $1500.

$1500 = 15(100)$

$1500 = 1500$

Practice

1. Rewrite using function notation: $y = \frac{5}{6}x - 2$.
2. Rewrite using function notation: $m = n^2 + 2n - 3$.
3. What is one benefit of using function notation?
4. Write a function that expresses the money earned after working some number of hours for $10 an hour.

5. Write a function that represents the number of cuts you need to cut a ribbon in x number of pieces.
6. Jackie and Mayra each will collect a $2 pledge for every basket they make during a game. Write two functions, one for each girl, expressing how much money she will collect.

Mixed Review

7. Compare the following numbers 23 __ 21.999.
8. Write an equation to represent the following: the quotient of 96 and 4 is g.
9. Write an inequality to represent the following: 11 minus b is at least 77.
10. Find the value of the variable k : $13(k) = 169$.