Linear Equations in Point-Slope Form

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CONCEPT 1

Linear Equations in Point-Slope Form

Here you'll learn how to work with linear equations and functions in point-slope form.

Suppose that the cost of a wedding was a function of the number of guests attending. If you knew the slope of the function and you also knew how much the wedding would cost if 150 guests attended, could you write a linear equation representing this situation? If so, what form of the equation would be easiest to use? In this Concept, you'll learn about the point-slope form of a linear equation so that you can answer questions like these.

Guidance

Equations can be written in many forms. The previous Concepts taught you how to write equations of lines in slope-intercept form. This Concept will provide a second way to write an equation of a line: **point-slope form**.

The equation of the line between any two points (x_1, y_1) and (x_2, y_2) can be written in the following form: $y - y_1 = m(x - x_1)$.

To write an equation in point-slope form, you need two things:

1. The slope of the line
2. A point on the line

Example A

Write an equation for a line containing (9, 3) and (4, 5).

Solution: Begin by finding the slope.

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{4-9} = -\frac{2}{5}$$

Instead of trying to find b (the $y-$intercept), you will use the point-slope formula.

$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{-2}{5}(x - 9)$$

It doesn't matter which point you use.

You could also use the other ordered pair to write the equation:

$$y - 5 = \frac{-2}{5}(x - 4)$$

These equations may look completely different, but by solving each one for y, you can compare the slope-intercept form to check your answer.

$$y-3=\frac{-2}{5}(x-9)\Rightarrow y=\frac{-2}{5}x+\frac{18}{5}+3$$
$$y=\frac{-2}{5}x+\frac{33}{5}$$
$$y-5=\frac{-2}{5}(x-4)$$
$$y=\frac{-2}{5}x+\frac{8}{5}+5$$
$$y=\frac{-2}{5}x+\frac{33}{5}$$

This process is called **rewriting in slope-intercept form.**

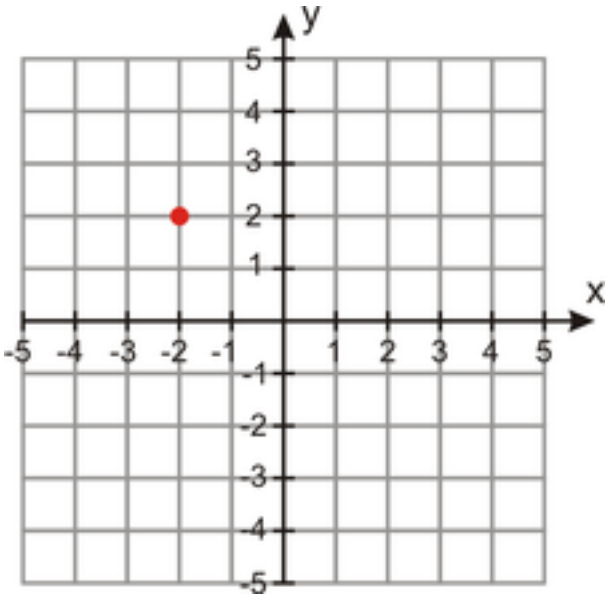
Graphing Equations Using Point-Slope Form

If you are given an equation in point-slope form, it is not necessary to re-write it in slope-intercept form in order to graph it. The point-slope form of the equation gives you enough information so you can graph the line.

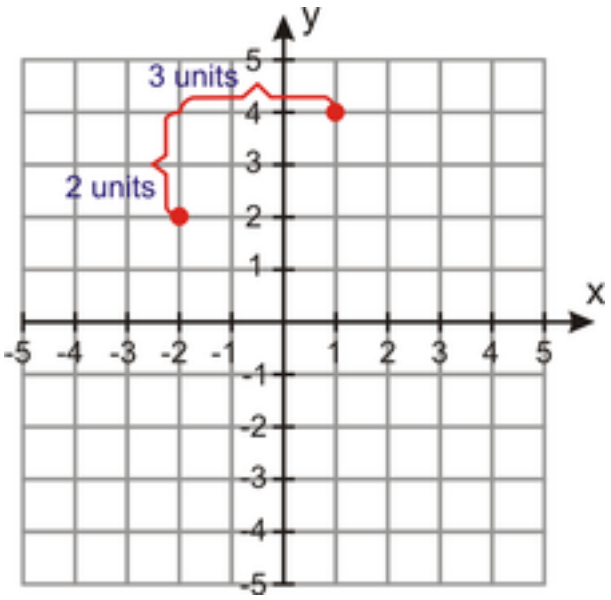
Example B

Make a graph of the line given by the equation $y-2=\frac{2}{3}(x+2)$.

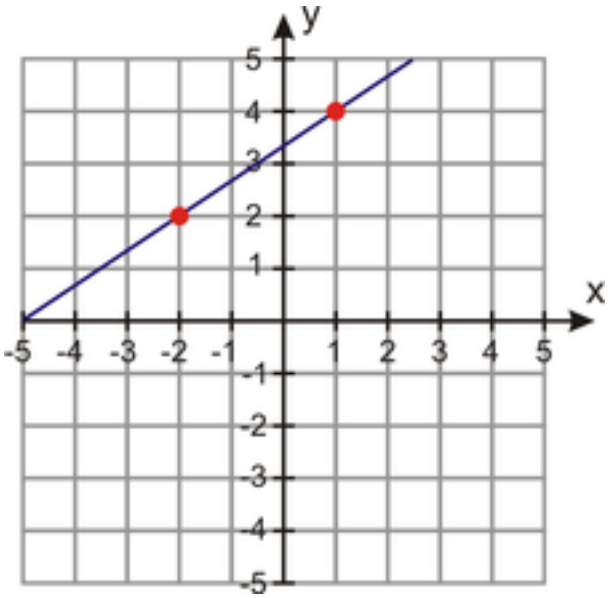
Solution: Begin by rewriting the equation to make it point-slope form: $y-2=\frac{2}{3}(x-(-2))$ Now we see that point (–2, 2) is on the line and that the slope $=\frac{2}{3}$. First plot point (–2, 2) on the graph.



A slope of $\frac{2}{3}$ tells you that from your point you should move 2 units up and 3 units to the right and draw another point.



Now draw a line through the two points and extend the line in both directions.



Writing a Linear Function in Point-Slope Form

Remember from the previous Concept that $f(x)$ and y are used interchangeably. Therefore, to write a function in point-slope form, you replace $y - y_1$ with $f(x) - y_1$.

Example C

Write the equation of the linear function in point-slope form.

$m = 9.8$ and $f(5.5) = 12.5$

Solution: This function has a slope of 9.8 and contains the ordered pair (5.5, 12.5). Substituting the appropriate values into point-slope form, we get the following:

$$y - 12.5 = 9.8(x - 5.5)$$

Replacing $y - y_1$ with $f(x) - y_1$, the equation in point-slope form is:

$$\begin{aligned} f(x) - 12.5 &= 9.8(x - 5.5) \\ f(x) - 12.5 &= 9.8x - 53.9 \\ f(x) &= 9.8x - 41.4 \end{aligned}$$

where the last equation is in slope-intercept form.

Vocabulary

Rewriting in slope-intercept form: When a linear equation is in any form, we can always isolate y to get the equation into slope-intercept form. This process is called ***rewriting in slope-intercept form.***

Guided Practice

Rewrite $y - 5 = 3(x - 2)$ in slope-intercept form.

Solution: Use the Distributive Property to simplify the right side of the equation:

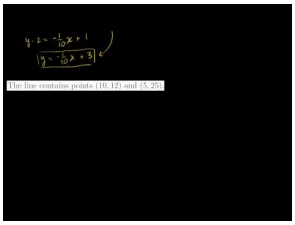
$$y - 5 = 3x - 6$$

Solve for y:

$$\begin{aligned} y - 5 + 5 &= 3x - 6 + 5 \\ y &= 3x - 1 \end{aligned}$$

Practice

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both. CK-12 Basic Algebra:Linear Equations inPoint-SlopeForm (9:38)



MEDIA

Click image to the left for more content.

1. What is the equation for a line containing the points (x_1, y_1) and (x_2, y_2) in point-slope form?
2. In what ways is it easier to use point-slope form rather than slope-intercept form?

In 3 - 13, write the equation for the line in point-slope form.

3. The slope is $\frac{1}{3}$; the $y-$intercept is –4.
4. The slope is $-\frac{1}{10}$ and contains the point (10, 2).
5. The slope is –75 and contains the point (0, 125).
6. The slope is 10 and contains the point (8, –2).
7. The line contains the points (–2, 3) and (–1, –2).
8. The line contains the points (0, 0) and (1, 2).
9. The line contains the points (10, 12) and (5, 25).
10. The line contains the points (2, 3) and (0, 3).
11. The line has a slope of $\frac{3}{5}$ and a $y-$intercept of –3.
12. The line has a slope of –6 and a $y-$intercept of 0.5.
13. The line contains the points (–4, –2) and (8, 12).

In 14 - 17, write each equation in slope-intercept form.

14. $y-2=3(x-1)$
15. $y+4=\frac{-2}{3}(x+6)$
16. $0=x+5$
17. $y=\frac{1}{4}(x-24)$

In 18 – 25, write the equation of the linear function in point-slope form.

18. $m=-\frac{1}{5}$ and $f(0)=7$
19. $m=-12$ and $f(-2)=5$
20. $f(-7)=5$ and $f(3)=-4$
21. $f(6)=0$ and $f(0)=6$
22. $m=3$ and $f(2)=-9$
23. $m=-\frac{9}{5}$ and $f(0)=32$
24. $m=25$ and $f(0)=250$
25. $f(32)=0$ and $f(77)=25$