Slope

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CONCEPT 1

Slope

Here you'll learn how to analyze vertical change and horizontal change so that you can calculate the slope of a line.

Suppose you have a toy airplane, and upon takeoff, it rises 5 feet for every 6 feet that it travels along the horizontal. What would be the slope of its ascent? Would it be a positive value or a negative value? In this Concept, you'll learn how to determine the slope of a line by analyzing vertical change and horizontal change so that you can handle problems such as this one.

Guidance

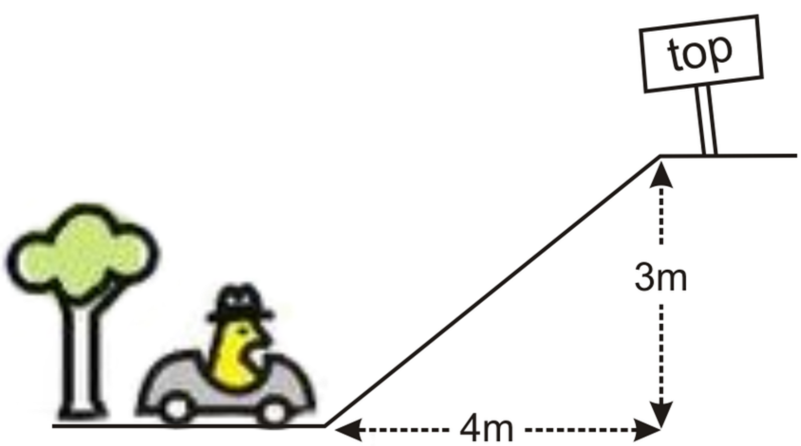
The pitch of a roof, the slant of a ladder against a wall, the incline of a road, and even your treadmill incline are all examples of slope.

The **slope** of a line measures its steepness (either negative or positive).

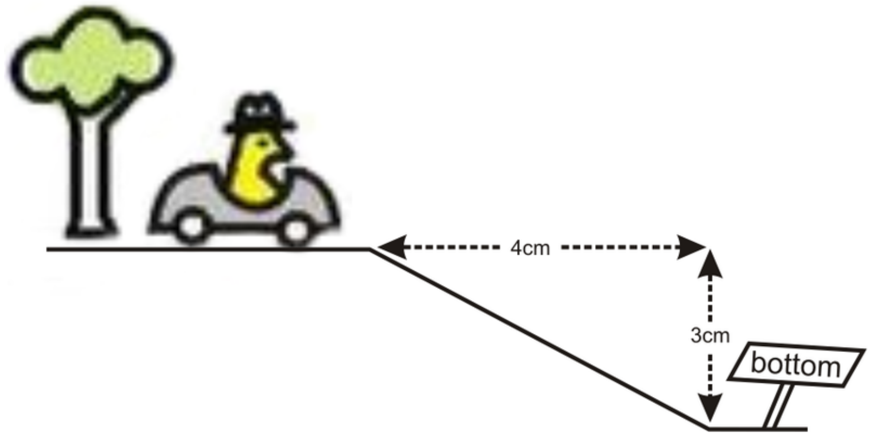
For example, if you have ever driven through a mountain range, you may have seen a sign stating, "10% incline." The percent tells you how steep the incline is. You have probably seen this on a treadmill too. The incline on a treadmill measures how steep you are walking uphill. Below is a more formal definition of slope.

The **slope** of a line is the vertical change divided by the horizontal change.

In the figure below, a car is beginning to climb up a hill. The height of the hill is 3 meters and the length of the hill is 4 meters. Using the definition above, the slope of this hill can be written as $\frac{3 \ meters}{4 \ meters} = \frac{3}{4}$. Because $\frac{3}{4} = 75\%$, we can say this hill has a 75% positive slope.



Similarly, if the car begins to descend *down* a hill, you can still determine the slope.



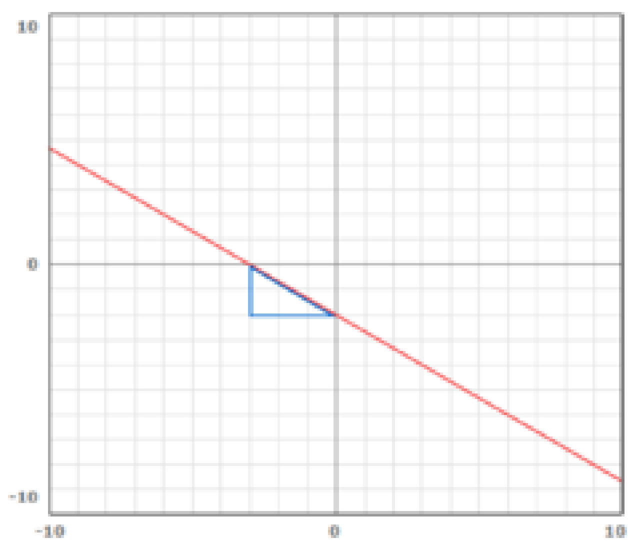
$$Slope = \frac{vertical\ change}{horizontal\ change} = \frac{-3}{4}$$

The slope in this instance is negative because the car is traveling downhill.

Another way to think of slope is: $slope = \frac{rise}{run}$.

When graphing an equation, slope is a very powerful tool. It provides the directions on how to get from one ordered pair to another. To determine slope, it is helpful to draw a ***slope-triangle***.

Using the following graph, choose two ordered pairs that have integer values such as (–3, 0) and (0, –2). Now draw in the slope triangle by connecting these two points as shown.



The vertical leg of the triangle represents the *rise* of the line and the horizontal leg of the triangle represents the *run* of the line. A third way to represent slope is:

$$slope = \frac{rise}{run}$$

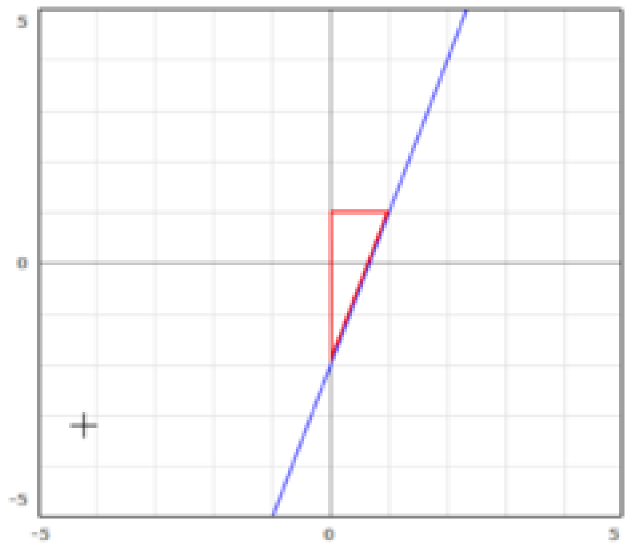
Starting at the left-most coordinate, count the number of vertical units and horizontal units it took to get to the right-most coordinate.

$$slope = \frac{rise}{run} = \frac{-2}{+3} = -\frac{2}{3}$$

Example A

Find the slope of the line graphed below.

Solution: Begin by finding two pairs of ordered pairs with integer values: (1, 1) *and* (0, –2).



Draw in the slope triangle.

Count the number of vertical units to get from the left ordered pair to the right.

Count the number of horizontal units to get from the left ordered pair to the right.

$$Slope = \frac{rise}{run} = \frac{+3}{+1} = \frac{3}{1}$$

A more algebraic way to determine a slope is by using a formula. The formula for slope is:

The slope between any two points (x_1, y_1) and (x_2, y_2) is: $slope = \frac{y_2 - y_1}{x_2 - x_1}$.

(x_1, y_1) represents one of the two ordered pairs and (x_2, y_2) represents the other. The following example helps show this formula.

Example B

Using the slope formula, determine the slope of the equation graphed in Example A.

Solution: Use the integer ordered pairs used to form the slope triangle: (1, 1) *and* (0, –2). Since (1, 1) is written first, it can be called (x_1, y_1). That means $(0, -2) = (x_2, y_2)$

Use the formula: $slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-1}{0-1} = \frac{-3}{-1} = \frac{3}{1}$

As you can see, the slope is the same regardless of the method you use. If the ordered pairs are fractional or spaced very far apart, it is easier to use the formula than to draw a slope triangle.

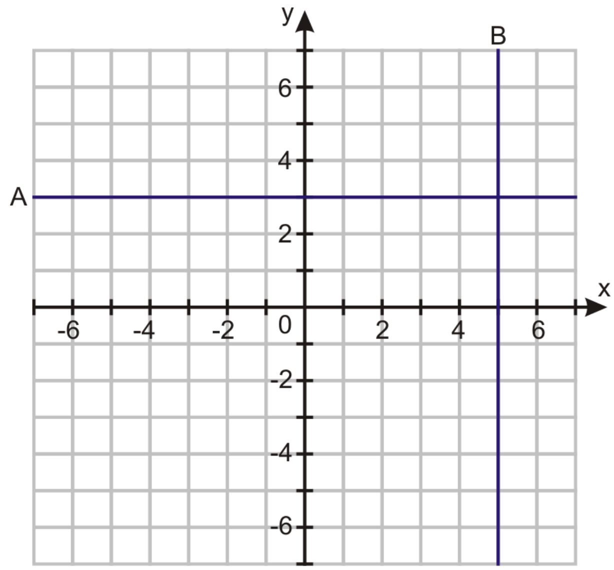
Types of Slopes

Slopes come in four different types: negative, zero, positive, and undefined. The first graph of this Concept had a negative slope. The second graph had a positive slope. Slopes with zero slopes are lines without any steepness, and undefined slopes cannot be computed.

Any line with a slope of zero will be a **horizontal** line with equation $y = some\ number$.

Any line with an undefined slope will be a **vertical** line with equation $x = some\ number$.

We will use the next two graphs to illustrate the previous definitions.



Example C

To determine the slope of *line A*, you need to find two ordered pairs with integer values.

(–4, 3) *and* (1, 3). Choose one ordered pair to represent (x_1, y_1) and the other to represent (x_2, y_2).

Now apply the formula: $slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-3}{1-(-4)} = \frac{0}{1+4} = 0$.

To determine the slope of *line B*, you need to find two ordered pairs on this line with integer values and apply the formula.

(5, 1) *and* (5, –6)

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-1}{5-5} = \frac{-7}{0} = Undefined$$

It is impossible to divide by zero, so the slope of *line B* cannot be determined and is called **undefined**.

Vocabulary

Slope: The ***slope*** of a line is the vertical change divided by the horizontal change. The ***slope*** of a line measures its steepness (either negative or positive).

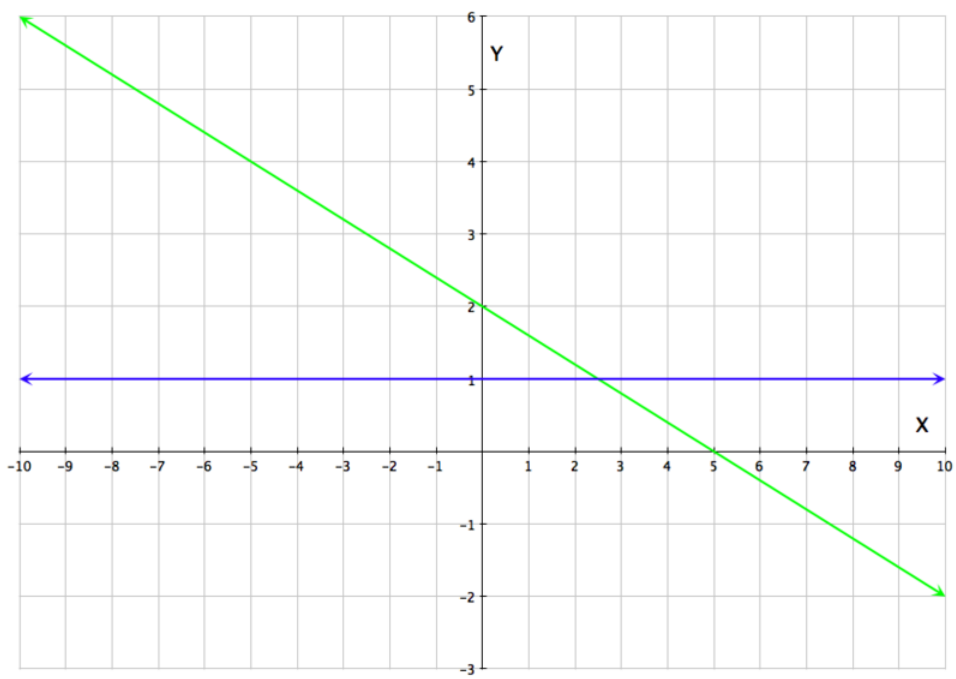
Slope-triangle: The ***slope-triangle*** is a right triangle whose hypotenuse is a portion of the graphed line, and where the two legs are vertical and horizontal segments whose lengths are used to calculate the slope.

Zero slope: A line with ***zero slope*** is a line without any steepness, or a horizontal line.

Undefined slope: An ***undefined slope*** cannot be computed. Vertical lines have undefined slopes.

Guided Practice

Find the slope of each line in the graph below:



Solution:

For each line, identify two coordinate pairs on the line and use them to calculate the slope.

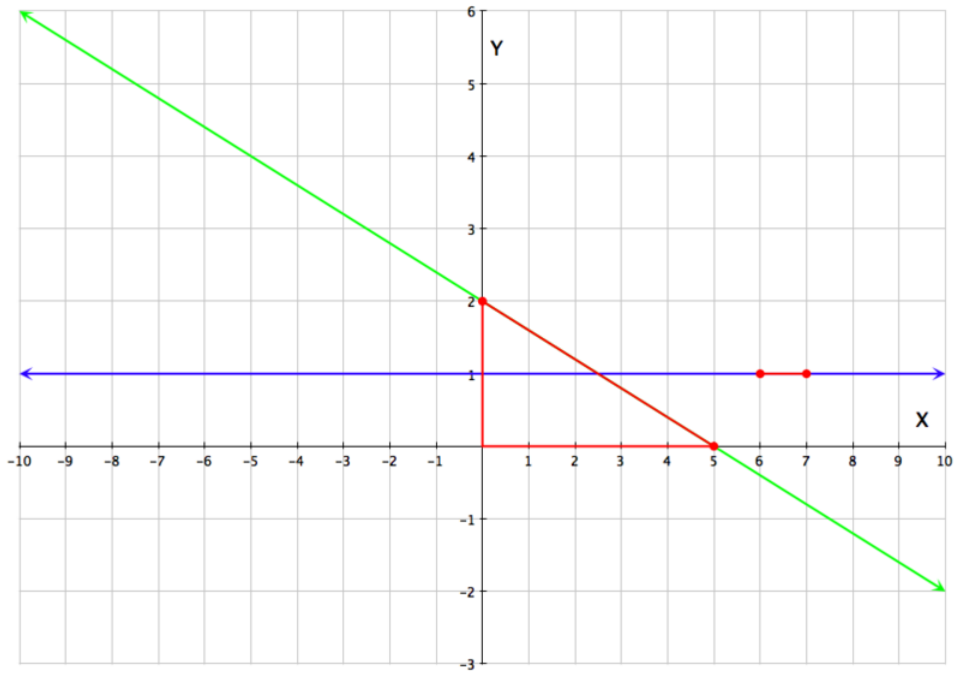
For the green line, one choice is $(0,2)$ and $(5,0)$. This results in a slope of:

slope $= \frac{0-2}{5-0} = -\frac{2}{5}$

For the blue line, one choice is $(6,1)$ and $(7,1)$. This results in a slope of:

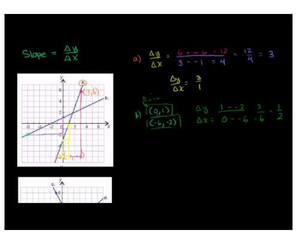
slope $= \frac{1-1}{7-6} = \frac{0}{1} = 0$

The slopes can be seen in this graph:



Practice

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both. CK-12 Basic Algebra:Slope and Rate of Change (13:42)

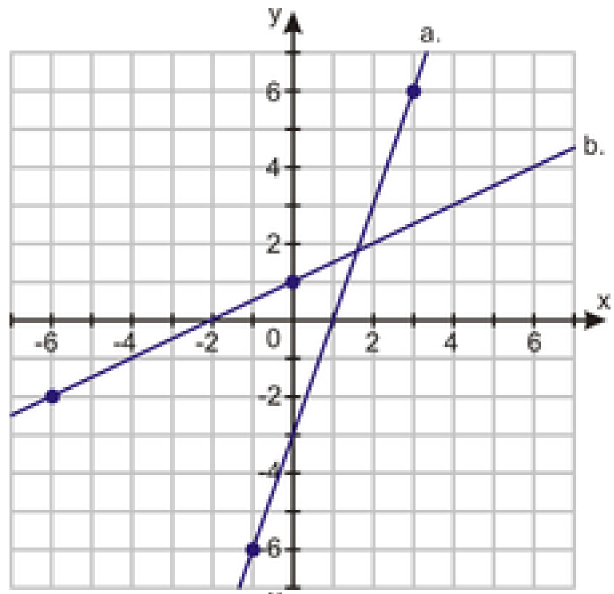


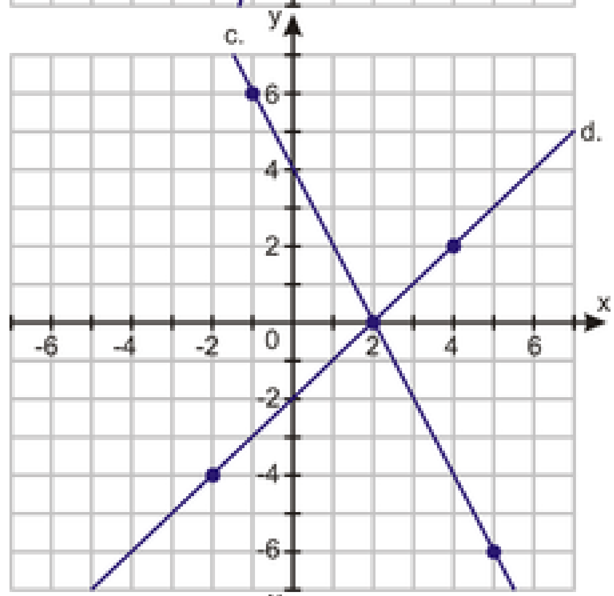
MEDIA

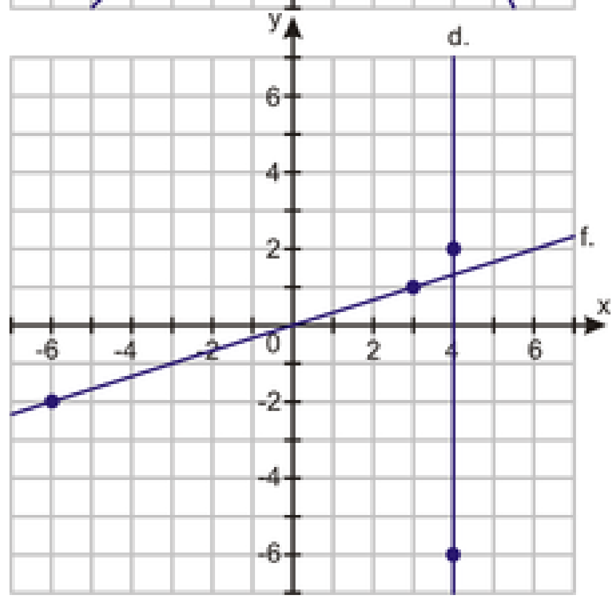
Click image to the left for more content.

1. Define *slope*.
2. Describe the two methods used to find slope. Which one do you prefer and why?
3. What is the slope of all vertical lines? Why is this true?
4. What is the slope of all horizontal lines? Why is this true?

Using the graphed coordinates, find the slope of each line.

6. 

7. 

8. 

In 9 – 21, find the slope between the two given points.

9. (–5, 7) and (0, 0)
10. (–3, –5) and (3, 11)
11. (3, –5) and (–2, 9)
12. (–5, 7) and (–5, 11)
13. (9, 9) and (–9, –9)
14. (3, 5) and (–2, 7)
15. $\left(\frac{1}{2}, \frac{3}{4}\right)$ and (–2, 6)
16. (–2, 3) and (4, 8)
17. (–17, 11) and (4, 11)
18. (31, 2) and (31, –19)
19. (0, –3) and (3, –1)
20. (2, 7) and (7, 2)
21. (0, 0) and $\left(\frac{2}{3}, \frac{1}{4}\right)$
22. Determine the slope of $y = 16$.
23. Determine the slope of $x = -99$.