Ratios and Proportions

Ratios and **proportions** have a fundamental place in mathematics. They are used in geometry, size changes, and trigonometry. This lesson expands upon the idea of fractions to include ratios and proportions.

A **ratio** is a fraction comparing two things with the same units.

A **rate** is a fraction comparing two things with different units.

You have experienced rates many times: 65 *mi/hour*, \$1.99/*pound*, \$3.79/yd^2. You have also experienced ratios. A "student to teacher" ratio shows approximately how many students one teacher is responsible for in a school.

Example 1: *The State Dining Room in the White House measures approximately 48 feet long by 36 feet wide. Compare the length of the room to the width, and express your answer as a ratio.*

Solution:

$$\frac{48\ feet}{36\ feet}=\frac{4}{3}$$

The length of the State Dining Room is $\frac{4}{3}$ the width.

A **proportion** is a statement in which two fractions are equal: $\frac{a}{b}=\frac{c}{d}$.

Example 2: *Is* $\frac{2}{3}=\frac{6}{12}$ *a proportion?*

Solution: Find the least common multiple of 3 and 12 to create a common denominator.

$$\frac{2}{3}=\frac{8}{12}\neq\frac{6}{12}$$

This is NOT a proportion because these two fractions are not equal.

A ratio can also be written using a colon instead of the fraction bar.

$\frac{a}{b}=\frac{c}{d}$ can also be read, "a is to b as c is to d" or $a:b=c:d$.

The values of a and d are called the **extremes** of the proportion and the values of b and c are called the **means**. To solve a proportion, you can use the **cross products**.

The **Cross Products of a Proportion:**

If $\frac{a}{b}=\frac{c}{d}$, then $ad=bc$.

Example 3: *Solve* $\frac{a}{9}=\frac{7}{6}$.

Solution: Apply the Cross Products of a Proportion.

$6a = 7(9)$

$6a = 63$

Solve for a.

$a = 10.5$

Consider the following situation: *A train travels at a steady speed. It covers 15 miles in 20 minutes. How far will it travel in 7 hours, assuming it continues at the same rate?* This is an example of a problem that can be solved using several methods, including proportions.

To solve using a proportion, you need to translate the statement into an algebraic sentence. The key to writing correct proportions is to keep the units the same in each fraction.

$$\frac{miles}{time} = \frac{miles}{time} \qquad \frac{miles}{time} \neq \frac{time}{miles}$$

You will be asked to solve this problem in the practice set.

Practice Set

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both. CK-12 Basic Algebra: Ratio and Proportion (10:25)

Write the following comparisons as ratios. Simplify fractions where possible.

1. \$150 to \$3
2. 150 boys to 175 girls
3. 200 minutes to 1 hour
4. 10 days to 2 weeks

In 5 – 10, write the ratios as a unit rate.

5. 54 hotdogs to 12 minutes
6. 5000 lbs to 250 in^2
7. 20 computers to 80 students
8. 180 students to 6 teachers
9. 12 meters to 4 floors
10. 18 minutes to 15 appointments
11. Give an example of a proportion that uses the numbers 5, 1, 6, and 30
12. In the following proportion, identify the means and the extremes: $\frac{5}{12} = \frac{35}{84}$

In 13 – 23, solve the proportion.

13. $\frac{13}{6} = \frac{5}{x}$
14. $\frac{1.25}{7} = \frac{3.6}{x}$
15. $\frac{6}{19} = \frac{x}{11}$
16. $\frac{1}{x} = \frac{0.01}{5}$
17. $\frac{300}{4} = \frac{x}{99}$
18. $\frac{2.75}{9} = \frac{x}{\left(\frac{2}{9}\right)}$

19. $\frac{1.3}{4} = \frac{x}{1.3}$
20. $\frac{0.1}{1.01} = \frac{1.9}{x}$
21. $\frac{5p}{12} = \frac{3}{11}$
22. $-\frac{9}{x} = \frac{4}{11}$
23. $\frac{n+1}{11} = -2$
24. A restaurant serves 100 people per day and takes in $908. If the restaurant were to serve 250 people per day, what might the cash collected be?
25. The highest mountain in Canada is Mount Yukon. It is $\frac{298}{67}$ the size of Ben Nevis, the highest peak in Scotland. Mount Elbert in Colorado is the highest peak in the Rocky Mountains. Mount Elbert is $\frac{220}{67}$ the height of Ben Nevis and $\frac{44}{48}$ the size of Mont Blanc in France. Mont Blanc is 4800 meters high. How high is Mount Yukon?
26. At a large high school, it is estimated that two out of every three students have a cell phone, and one in five of all students have a cell phone that is one year old or less. Out of the students who own a cell phone, what proportion own a phone that is more than one year old?
27. The price of a Harry Potter Book on Amazon.com is $10.00. The same book is also available used for $6.50. Find two ways to compare these prices.
28. To prepare for school, you purchased 10 notebooks for $8.79. How many notebooks can you buy for $5.80?
29. It takes 1 cup mix and  cup water to make 6 pancakes. How much water and mix is needed to make 21 pancakes?
30. Ammonia is a compound consisting of a 1:3 ratio of nitrogen and hydrogen atoms. If a sample contains 1,983 hydrogen atoms, how many nitrogen atoms are present?
31. The Eagles have won 5 out of their last 9 games. If this trend continues, how many games will they have won in the 63-game season?
32. Solve the train situation described earlier in this lesson.

Mixed Review

33. Solve $\frac{15}{16} \div \frac{5}{8}$.
34. Evaluate $|9 - 108|$.
35. Simplify: $8(8 - 3x) - 2(1 + 8x)$.
36. Solve for $n: \ 7(n + 7) = -7$.
37. Solve for $x: \ -22 = -3 + x$.
38. Solve for $u: \ 18 = 2u$.
39. Simplify: $-\frac{1}{7} - \left(-1\frac{1}{3}\right)$.
40. Evaluate: $5 \times \frac{p}{6}|n|$ when $n = 10$ *and* $p = -6$.
41. Make a table for $-4 \le x \le 4$ *for* $f(x) = \frac{1}{8}x + 2$.
42. Write as an English phrase: $y + 11$.

Scale and Indirect Measurement

We are occasionally faced with having to make measurements of things that would be difficult to measure directly: the height of a tall tree, the width of a wide river, the height of the moon's craters, even the distance between two cities separated by mountainous terrain. In such circumstances, measurements can be made