Ratios and Rates: Proportions*

Wade Ellis
Denny Burzynski

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Abstract

This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses proportions. By the end of the module students should be able to describe proportions and find the missing factor in a proportion and be able to work with proportions involving rates.

1 Section Overview

- Ratios, Rates, and Proportions
- Finding the Missing Factor in a Proportion
- Proportions Involving Rates

2 Ratios, Rates, and Proportions

Ratio, Rate
We have defined a **ratio** as a comparison, by division, of two pure numbers or two *like* denominate numbers.
We have defined a **rate** as a comparison, by division, of two *unlike* denominate numbers.
Proportion
A **proportion** is a statement that two ratios or rates are equal. The following two examples show how to read proportions.

$$\frac{3}{4} = \frac{6}{8}$$

3 is to 4 as 6 is to 8

$$\frac{25 \text{ miles}}{1 \text{ gallon}} = \frac{50 \text{ miles}}{2 \text{ gallons}}$$

25 miles is to 1 gallon as 50 miles is to 2 gallons

2.1 Sample Set A

Write or read each proportion.

Example 1
$\frac{3}{5} = \frac{12}{20}$

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3 is to 5 as 12 is to 20

Example 2
$\frac{10 \text{ items}}{5 \text{ dollars}} = \frac{2 \text{ items}}{1 \text{ dollar}}$
10 items is to 5 dollars as 2 items is to 1 dollar

Example 3
8 is to 12 as 16 is to 24.
$\frac{8}{12} = \frac{16}{24}$

Example 4
50 milligrams of vitamin C is to 1 tablet as 300 milligrams of vitamin C is to 6 tablets.
$\frac{50}{1} = \frac{300}{6}$

2.2 Practice Set A

Write or read each proportion.

Exercise 1 ***(Solution on p. 9.)***
$\frac{3}{8} = \frac{6}{16}$

Exercise 2 ***(Solution on p. 9.)***
$\frac{2 \text{ people}}{1 \text{ window}} = \frac{10 \text{ people}}{5 \text{ windows}}$

Exercise 3 ***(Solution on p. 9.)***
15 is to 4 as 75 is to 20.

Exercise 4 ***(Solution on p. 9.)***
2 plates are to 1 tray as 20 plates are to 10 trays.

3 Finding the Missing Factor in a Proportion

Many practical problems can be solved by writing the given information as proportions. Such proportions will be composed of three specified numbers and one unknown number. It is customary to let a letter, such as x, represent the unknown number. An example of such a proportion is
$\frac{x}{4} = \frac{20}{16}$
This proportion is read as " x is to 4 as 20 is to 16."

There is a method of solving these proportions that is based on the equality of fractions. Recall that two fractions are equivalent if and only if their cross products are equal. For example,

$$\frac{3}{4} = \frac{6}{8} \quad \text{since} \quad \frac{3}{4} \times \frac{6}{8}$$
$$3 \cdot 8 = 6 \cdot 4$$
$$24 = 24$$

Notice that in a proportion that contains three specified numbers and a letter representing an unknown quantity, that regardless of where the letter appears, the following situation always occurs.
$\underbrace{(\text{number}) \cdot (\text{letter}) = (\text{number}) \cdot (\text{number})}$

We recognize this as a multiplication statement. Specifically, it is a missing factor statement. (See here[1] for a discussion of multiplication statements.) For example,

[1] "Introduction to Fractions and Multiplication and Division of Fractions: Applications Involving Fractions" <http://cnx.org/content/m34930/latest/>

$\frac{x}{4} = \frac{20}{16}$ means that $16 \cdot x = 4 \cdot 20$
$\frac{4}{x} = \frac{16}{20}$ means that $4 \cdot 20 = 16 \cdot x$
$\frac{5}{4} = \frac{x}{16}$ means that $5 \cdot 16 = 4 \cdot x$
$\frac{5}{4} = \frac{20}{x}$ means that $5 \cdot x = 4 \cdot 20$

Each of these statements is a multiplication statement. Specifically, each is a missing factor statement. (The letter used here is x, whereas M was used in here[2].)

Finding the Missing Factor in a Proportion

The missing factor in a missing factor statement can be determined by dividing the product by the known factor, that is, if x represents the missing factor, then
$x =$ (product) $\div$ (known factor)

3.1 Sample Set B

Find the unknown number in each proportion.

Example 5

$\frac{x}{4} = \frac{20}{16}$. Find the cross product.

$$\begin{aligned} 16 \cdot x &= 20 \cdot 4 \\ 16 \cdot x &= 80 \quad \text{Divide the product 80 by the known factor 16.} \\ x &= \frac{80}{16} \\ x &= 5 \quad \text{The unknown number is 5.} \end{aligned}$$

This mean that $\frac{5}{4} = \frac{20}{16}$, or 5 is to 4 as 20 is to 16.

Example 6

$\frac{5}{x} = \frac{20}{16}$. Find the cross product.

$$\begin{aligned} 5 \cdot 16 &= 20 \cdot x \\ 80 &= 20 \cdot x \quad \text{Divide the product 80 by the known factor 20.} \\ \frac{80}{20} &= x \\ 4 &= x \quad \text{The unknown number is 4.} \end{aligned}$$

This means that $\frac{5}{4} = \frac{20}{16}$, or, 5 is to 4 as 20 is to 6.

Example 7

$\frac{16}{3} = \frac{64}{x}$ Find the cross product.

$$\begin{aligned} 16 \cdot x &= 64 \cdot 3 \\ 16 \cdot x &= 192 \quad \text{Divide 192 by 16.} \\ x &= \frac{192}{16} \\ x &= 12 \quad \text{The unknown number is 12.} \end{aligned}$$

The means that $\frac{16}{3} = \frac{64}{12}$, or, 16 is to 3 as 64 is to 12.

Example 8

$\frac{9}{8} = \frac{x}{40}$ Find the cross products.

[2] "Introduction to Fractions and Multiplication and Division of Fractions: Applications Involving Fractions" <http://cnx.org/content/m34930/latest/>

$$\begin{aligned} 9 \cdot 40 &= 8 \cdot x \\ 360 &= 8 \cdot x \qquad \text{Divide 360 by 8.} \\ \frac{360}{8} &= x \\ 45 &= x \qquad \text{The unknown number is 45.} \end{aligned}$$

3.2 Practice Set B

Find the unknown number in each proportion.

Exercise 5 *(Solution on p. 9.)*

$\frac{x}{8} = \frac{12}{32}$

Exercise 6 *(Solution on p. 9.)*

$\frac{7}{x} = \frac{14}{10}$

Exercise 7 *(Solution on p. 9.)*

$\frac{9}{11} = \frac{x}{55}$

Exercise 8 *(Solution on p. 9.)*

$\frac{1}{6} = \frac{8}{x}$

4 Proportions Involving Rates

Recall that a rate is a comparison, by division, of unlike denominate numbers. We must be careful when setting up proportions that involve rates. The *form* is important. For example, if a rate involves two types of units, say unit type 1 and unit type 2, we can write

$$\frac{\text{unit type 1}}{\text{unit type 2}} = \frac{\text{unit type 1}}{\text{unit type 2}}$$

Same units appear on same side.

Same units appear on same side.

or

$$\frac{\text{unit type 1}}{\text{unit type 1}} = \frac{\text{unit type 2}}{\text{unit type 2}}$$

Same units appear on same side. Same units appear on same side.

Both cross products produce a statement of the type
(unit type 1) · (unit type 2) = (unit type 1) · (unit type 2)
which we take to mean the comparison

$$\underbrace{\underbrace{\text{(unit type 1) is to (unit type 2)}}_{\text{Comparison of type 1 with type 2}} \quad \text{as} \quad \underbrace{\text{(unit type 1) is to (unit type 2)}}_{\text{Comparison of type 1 with type 2}}}_{\text{Same overall type}}$$

Examples of correctly expressed proportions are the following:

1. $\frac{\text{mi}}{\text{hr}} = \frac{\text{mi}}{\text{hr}}$ Same units appear on the same side
2. $\frac{\text{mi}}{\text{mi}} = \frac{\text{hr}}{\text{hr}}$

Same units appear on the same side.

However, if we write the same type of units on different sides, such as,
$\frac{\text{unit type 1}}{\text{unit type 2}} = \frac{\text{unit type 2}}{\text{unit type 1}}$
the cross product produces a statement of the form

$$\underbrace{\underbrace{(\text{unit type 1}) \cdot (\text{unit type 1})}_{\text{Comparison of type 1 with type 1}} = \underbrace{(\text{unit type 2}) \cdot (\text{unit type 2})}_{\text{Comparison of type 2 with type 2}}}_{\text{Different overall types}}$$

We can see that this is an incorrect comparison by observing the following example: It is *incorrect* to write
$\frac{\text{2 hooks}}{\text{3 poles}} = \frac{\text{6 poles}}{\text{4 hooks}}$
for two reason.

1. The cross product is numerically wrong: $(2 \cdot 4 \neq 3 \cdot 6)$.
2. The cross product produces the statement "hooks are to hooks as poles are to poles," which makes no sense.

5 Exercises

Exercise 9 *(Solution on p. 9.)*

A statement that two ratios or _____ are equal is called a _____.

For the following 9 problems, write each proportion in fractional form.

Exercise 10

3 is to 7 as 18 is to 42.

Exercise 11 *(Solution on p. 9.)*

1 is to 11 as 3 is to 33.

Exercise 12

9 is to 14 as 27 is to 42.

Exercise 13 *(Solution on p. 9.)*

6 is to 90 as 3 is to 45.

Exercise 14

5 liters is to 1 bottle as 20 liters is to 4 bottles.

Exercise 15 *(Solution on p. 9.)*

18 grams of cobalt is to 10 grams of silver as 36 grams of cobalt is to 20 grams of silver.

Exercise 16

4 cups of water is to 1 cup of sugar as 32 cups of water is to 8 cups of sugar.

Exercise 17 *(Solution on p. 9.)*

3 people absent is to 31 people present as 15 people absent is to 155 people present.

Exercise 18

6 dollars is to 1 hour as 90 dollars is to 15 hours.

For the following 10 problems, write each proportion as a sentence.

Exercise 19 *(Solution on p. 9.)*

$\frac{3}{4} = \frac{15}{20}$

Exercise 20

$\frac{1}{8} = \frac{5}{40}$

Exercise 21 *(Solution on p. 9.)*

$\frac{3 \text{ joggers}}{100 \text{ feet}} = \frac{6 \text{ joggers}}{200 \text{ feet}}$

Exercise 22

$\frac{12 \text{ marshmallows}}{3 \text{ sticks}} = \frac{36 \text{ marshmallows}}{9 \text{ sticks}}$

Exercise 23 *(Solution on p. 9.)*

$\frac{40 \text{ miles}}{80 \text{ miles}} = \frac{2 \text{ gallons}}{4 \text{ gallons}}$

Exercise 24

$\frac{4 \text{ couches}}{10 \text{ couches}} = \frac{2 \text{ houses}}{5 \text{ houses}}$

Exercise 25 *(Solution on p. 9.)*

$\frac{1 \text{ person}}{1 \text{ job}} = \frac{8 \text{ people}}{8 \text{ jobs}}$

Exercise 26

$\frac{1 \text{ popsicle}}{2 \text{ children}} = \frac{\frac{1}{2} \text{ popsicle}}{1 \text{ child}}$

Exercise 27 *(Solution on p. 9.)*

$\frac{2{,}000 \text{ pounds}}{1 \text{ ton}} = \frac{60{,}000 \text{ pounds}}{30 \text{ tons}}$

Exercise 28

$\frac{1 \text{ table}}{5 \text{ tables}} = \frac{2 \text{ people}}{10 \text{ people}}$

For the following 10 problems, solve each proportion.

Exercise 29 *(Solution on p. 9.)*

$\frac{x}{5} = \frac{6}{15}$

Exercise 30

$\frac{x}{10} = \frac{28}{40}$

Exercise 31 *(Solution on p. 9.)*

$\frac{5}{x} = \frac{10}{16}$

Exercise 32

$\frac{13}{x} = \frac{39}{60}$

Exercise 33 *(Solution on p. 9.)*

$\frac{1}{3} = \frac{x}{24}$

Exercise 34

$\frac{7}{12} = \frac{x}{60}$

Exercise 35 *(Solution on p. 9.)*

$\frac{8}{3} = \frac{72}{x}$

Exercise 36

$\frac{16}{1} = \frac{48}{x}$

Exercise 37 *(Solution on p. 9.)*

$\frac{x}{25} = \frac{200}{125}$

Exercise 38

$\frac{65}{30} = \frac{x}{60}$

For the following 5 problems, express each sentence as a proportion then solve the proportion.

Exercise 39 *(Solution on p. 9.)*
5 hats are to 4 coats as x hats are to 24 coats.

Exercise 40
x cushions are to 2 sofas as 24 cushions are to 16 sofas.

Exercise 41 *(Solution on p. 9.)*
1 spacecraft is to 7 astronauts as 5 spacecraft are to x astronauts.

Exercise 42
56 microchips are to x circuit boards as 168 microchips are to 3 circuit boards.

Exercise 43 *(Solution on p. 10.)*
18 calculators are to 90 calculators as x students are to 150 students.

Exercise 44
x dollars are to \$40,000 as 2 sacks are to 1 sack.

Indicate whether the proportion is true or false.

Exercise 45 *(Solution on p. 10.)*
$\frac{3}{16} = \frac{12}{64}$

Exercise 46
$\frac{2}{15} = \frac{10}{75}$

Exercise 47 *(Solution on p. 10.)*
$\frac{1}{9} = \frac{3}{30}$

Exercise 48
$\frac{6 \text{ knives}}{7 \text{ forks}} = \frac{12 \text{ knives}}{15 \text{ forks}}$

Exercise 49 *(Solution on p. 10.)*
$\frac{33 \text{ miles}}{1 \text{ gallon}} = \frac{99 \text{ miles}}{3 \text{ gallons}}$

Exercise 50
$\frac{320 \text{ feet}}{5 \text{ seconds}} = \frac{65 \text{ feet}}{1 \text{ second}}$

Exercise 51 *(Solution on p. 10.)*
$\frac{35 \text{ students}}{70 \text{ students}} = \frac{1 \text{ class}}{2 \text{ classes}}$

Exercise 52
$\frac{9 \text{ ml chloride}}{45 \text{ ml chloride}} = \frac{1 \text{ test tube}}{7 \text{ test tubes}}$

5.1 Exercises for Review

Exercise 53 *(Solution on p. 10.)*
(here[3]) Use the number 5 and 7 to illustrate the commutative property of addition.

Exercise 54
(here[4]) Use the numbers 5 and 7 to illustrate the commutative property of multiplication.

Exercise 55 *(Solution on p. 10.)*
(here[5]) Find the difference. $\frac{5}{14} - \frac{3}{22}$.

Exercise 56
(here[6]) Find the product. $8.06129 \cdot 1,000$.

[3] "Addition and Subtraction of Whole Numbers: Properties of Addition" <http://cnx.org/content/m34802/latest/>
[4] "Multiplication and Division of Whole Numbers: Summary of Key Concepts" <http://cnx.org/content/m34868/latest/>
[5] "Addition and Subtraction of Fractions, Comparing Fractions, and Complex Fractions: Addition and Subtraction of Fractions with Unlike Denominators" <http://cnx.org/content/m34935/latest/>
[6] "Decimals: Multiplication of Decimals" <http://cnx.org/content/m34963/latest/>

Exercise 57 ***(Solution on p. 10.)***

(here[7]) Write the simplified fractional form of the rate "sixteen sentences to two paragraphs."

[7] "Ratios and Rates: Ratios and Rates" <http://cnx.org/content/m34980/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 2)
3 is to 8 as 6 is to 16
Solution to Exercise (p. 2)
2 people are to 1 window as 10 people are to 5 windows
Solution to Exercise (p. 2)
$\frac{15}{4} = \frac{75}{20}$
Solution to Exercise (p. 2)
$\frac{2 \text{ plates}}{1 \text{ tray}} = \frac{20 \text{ plates}}{10 \text{ trays}}$
Solution to Exercise (p. 4)
$x = 3$
Solution to Exercise (p. 4)
$x = 5$
Solution to Exercise (p. 4)
$x = 45$
Solution to Exercise (p. 4)
$x = 48$
Solution to Exercise (p. 5)
rates, proportion
Solution to Exercise (p. 5)
$\frac{1}{11} = \frac{3}{33}$
Solution to Exercise (p. 5)
$\frac{6}{90} = \frac{3}{45}$
Solution to Exercise (p. 5)
$\frac{18 \text{ gr cobalt}}{10 \text{ gr silver}} = \frac{36 \text{ gr cobalt}}{20 \text{ gr silver}}$
Solution to Exercise (p. 5)
$\frac{3 \text{ people absent}}{31 \text{ people present}} = \frac{15 \text{ people absent}}{155 \text{ people present}}$
Solution to Exercise (p. 6)
3 is to 4 as 15 is to 20
Solution to Exercise (p. 6)
3 joggers are to 100 feet as 6 joggers are to 200 feet
Solution to Exercise (p. 6)
40 miles are to 80 miles as 2 gallons are to 4 gallons
Solution to Exercise (p. 6)
1 person is to 1 job as 8 people are to 8 jobs
Solution to Exercise (p. 6)
2,000 pounds are to 1 ton as 60,000 pounds are to 30 tons
Solution to Exercise (p. 6)
$x = 2$
Solution to Exercise (p. 6)
$x = 8$
Solution to Exercise (p. 6)
$x = 8$
Solution to Exercise (p. 6)
$x = 27$
Solution to Exercise (p. 6)
$x = 40$
Solution to Exercise (p. 6)
$x = 30$

Solution to Exercise (p. 7)

$x = 35$

Solution to Exercise (p. 7)

$x = 30$

Solution to Exercise (p. 7)

true

Solution to Exercise (p. 7)

false

Solution to Exercise (p. 7)

true

Solution to Exercise (p. 7)

true

Solution to Exercise (p. 7)

$5 + 7 = 12$

$7 + 5 = 12$

Solution to Exercise (p. 7)

$\frac{17}{77}$

Solution to Exercise (p. 8)

$\frac{8 \text{ sentences}}{1 \text{ paragraph}}$