

F DISTRIBUTION AND ANOVA: INTRODUCTION*

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Abstract

This module provides a brief introduction on handling hypothesis tests with two means by the one-way Analysis of Variance (ANOVA), F Distribution, and the Test of Two Variances statistical analysis.

1 Student Learning Objectives

By the end of this chapter, the student should be able to:

- Interpret the F probability distribution as the number of groups and the sample size change.
- Discuss two uses for the F distribution, ANOVA and the test of two variances.
- Conduct and interpret ANOVA.
- Conduct and interpret hypothesis tests of two variances (optional).

2 Introduction

Many statistical applications in psychology, social science, business administration, and the natural sciences involve several groups. For example, an environmentalist is interested in knowing if the average amount of pollution varies in several bodies of water. A sociologist is interested in knowing if the amount of income a person earns varies according to his or her upbringing. A consumer looking for a new car might compare the average gas mileage of several models.

For hypothesis tests involving more than two averages, statisticians have developed a method called "Analysis of Variance" (abbreviated ANOVA). In this chapter, you will study the simplest form of ANOVA called single factor or one-way ANOVA. You will also study the F distribution, used for ANOVA, and the test of two variances. This is just a very brief overview of ANOVA. You will study this topic in much greater detail in future statistics courses.

- ANOVA, as it is presented here, relies heavily on a calculator or computer.
- For further information about ANOVA, use the online link ANOVA¹. Use the back button to return here. (The url is http://en.wikipedia.org/wiki/Analysis_of_variance.)

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¹http://en.wikipedia.org/wiki/Analysis_of_variance

F DISTRIBUTION AND ANOVA: PURPOSE AND BASIC ASSUMPTION OF ANOVA*

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Abstract

This module describes the assumptions needed for implementing an ANOVA and how to set up the hypothesis test for the ANOVA.

1 F Distribution and ANOVA: Purpose and Basic Assumption of ANOVA

The purpose of an **ANOVA** test is to determine the existence of a statistically significant difference among several group means. The test actually uses **variances** to help determine if the means are equal or not.

In order to perform an ANOVA test, there are three basic **assumptions** to be fulfilled:

- Each population from which a sample is taken is assumed to be normal.
- Each sample is randomly selected and independent.
- The populations are assumed to have **equal standard deviations (or variances)**.

2 The Null and Alternate Hypotheses

The null hypothesis is simply that all the group population means are the same. The alternate hypothesis is that at least one pair of means is different. For example, if there are k groups:

$$H_o : \mu_1 = \mu_2 = \mu_3 = \dots = \mu_k$$

H_a : At least two of the group means $\mu_1, \mu_2, \mu_3, \dots, \mu_k$ are not equal.

Glossary

Definition 1: Analysis of Variance

Also referred to as ANOVA. A method of testing whether or not the means of three or more populations are equal. The method is applicable if:

- All populations of interest are normally distributed.
- The populations have equal standard deviations.

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- Samples (not necessarily of the same size) are randomly and independently selected from each population.

The test statistic for analysis of variance is the F-ratio.

Definition 2: Variance

Mean of the squared deviations from the mean. Square of the standard deviation.



F DISTRIBUTION AND ANOVA: THE F DISTRIBUTION AND THE F RATIO*

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Abstract

This module describes how to calculate the F Ratio and F Distribution based on the hypothesis test for the ANOVA.

The distribution used for the hypothesis test is a new one. It is called the F distribution, named after Sir Ronald Fisher, an English statistician. The F statistic is a ratio (a fraction). There are two sets of degrees of freedom; one for the numerator and one for the denominator.

For example, if F follows an F distribution and the degrees of freedom for the numerator are 4 and the degrees of freedom for the denominator are 10, then $F \sim F_{4,10}$.

To calculate the F ratio, two estimates of the variance are made.

1. **Variance between samples:** An estimate of σ^2 that is the variance of the sample means. If the samples are different sizes, the variance between samples is weighted to account for the different sample sizes. The variance is also called **variation due to treatment or explained variation**.
2. **Variance within samples:** An estimate of σ^2 that is the average of the sample variances (also known as a pooled variance). When the sample sizes are different, the variance within samples is weighted. The variance is also called the **variation due to error or unexplained variation**.

- SS_{between} = the sum of squares that represents the variation among the different samples.
- SS_{within} = the sum of squares that represents the variation within samples that is due to chance.

To find a "sum of squares" means to add together squared quantities which, in some cases, may be weighted. We used sum of squares to calculate the sample variance and the sample standard deviation in **Descriptive Statistics**.

MS means "mean square." MS_{between} is the variance between groups and MS_{within} is the variance within groups.

Calculation of Sum of Squares and Mean Square

- k = the number of different groups
- n_j = the size of the j th group
- s_j = the sum of the values in the j th group

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- N = total number of all the values combined. (total sample size: $\sum n_j$)
- x = one value: $\sum x = \sum s_j$
- Sum of squares of all values from every group combined: $\sum x^2$
- Between group variability: $SS_{\text{total}} = \sum x^2 - \frac{(\sum x)^2}{N}$
- Total sum of squares: $\sum x^2 - \frac{(\sum x)^2}{N}$
- Explained variation- sum of squares representing variation among the different samples $SS_{\text{between}} = \sum \left[\frac{(s_j)^2}{n_j} \right] - \frac{(\sum s_j)^2}{N}$
- Unexplained variation- sum of squares representing variation within samples due to chance: $SS_{\text{within}} = SS_{\text{total}} - SS_{\text{between}}$
- df's for different groups (df's for the numerator): $df_{\text{between}} = k - 1$
- Equation for errors within samples (df's for the denominator): $df_{\text{within}} = N - k$
- Mean square (variance estimate) explained by the different groups: $MS_{\text{between}} = \frac{SS_{\text{between}}}{df_{\text{between}}}$
- Mean square (variance estimate) that is due to chance (unexplained): $MS_{\text{within}} = \frac{SS_{\text{within}}}{df_{\text{within}}}$

MS_{between} and MS_{within} can be written as follows:

- $MS_{\text{between}} = \frac{SS_{\text{between}}}{df_{\text{between}}} = \frac{SS_{\text{between}}}{k-1}$
- $MS_{\text{within}} = \frac{SS_{\text{within}}}{df_{\text{within}}} = \frac{SS_{\text{within}}}{N-k}$

The ANOVA test depends on the fact that MS_{between} can be influenced by population differences among means of the several groups. Since MS_{within} compares values of each group to its own group mean, the fact that group means might be different does not affect MS_{within} .

The null hypothesis says that all groups are samples from populations having the same normal distribution. The alternate hypothesis says that at least two of the sample groups come from populations with different normal distributions. If the null hypothesis is true, MS_{between} and MS_{within} should both estimate the same value.

NOTE: The null hypothesis says that all the group population means are equal. The hypothesis of equal means implies that the populations have the same normal distribution because it is assumed that the populations are normal and that they have equal variances.

F-Ratio or F Statistic

$$F = \frac{MS_{\text{between}}}{MS_{\text{within}}} \tag{1}$$

If MS_{between} and MS_{within} estimate the same value (following the belief that H_o is true), then the F-ratio should be approximately equal to 1. Only sampling errors would contribute to variations away from 1. As it turns out, MS_{between} consists of the population variance plus a variance produced from the differences between the samples. MS_{within} is an estimate of the population variance. Since variances are always positive, if the null hypothesis is false, MS_{between} will be larger than MS_{within} . The F-ratio will be larger than 1.

The above calculations were done with groups of different sizes. If the groups are the same size, the calculations simplify somewhat and the F ratio can be written as:

F-Ratio Formula when the groups are the same size

$$F = \frac{n \cdot (s_{-x})^2}{(s_{\text{pooled}})} \tag{2}$$

where ...

- $(s_{-x})^2$ =the variance of the sample means
- n =the sample size of each group



- $(s_{\text{pooled}})^2$ = the mean of the sample variances (pooled variance)
- $df_{\text{numerator}} = k - 1$
- $df_{\text{denominator}} = k(n - 1) = N - k$

The ANOVA hypothesis test is always right-tailed because larger F-values are way out in the right tail of the F-distribution curve and tend to make us reject H_o .

1 Notation

The notation for the F distribution is $F \sim F_{df(\text{num}), df(\text{denom})}$

where $df(\text{num}) = df_{\text{between}}$ and $df(\text{denom}) = df_{\text{within}}$

The mean for the F distribution is $\mu = \frac{df(\text{num})}{df(\text{denom}) - 1}$

F DISTRIBUTION AND ANOVA: FACTS ABOUT THE F DISTRIBUTION*

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Abstract

This module states the factors associated with F Distributions and provides students with some examples to help further understand the concept. Students will be given the opportunity to see F Distributions in action through participation in an optional classroom exercise.

1. The curve is not symmetrical but skewed to the right.
2. There is a different curve for each set of dfs.
3. The F statistic is greater than or equal to zero.
4. As the degrees of freedom for the numerator and for the denominator get larger, the curve approximates the normal.
5. Other uses for the F distribution include comparing two variances and Two-Way Analysis of Variance. Comparing two variances is discussed at the end of the chapter. Two-Way Analysis is mentioned for your information only.

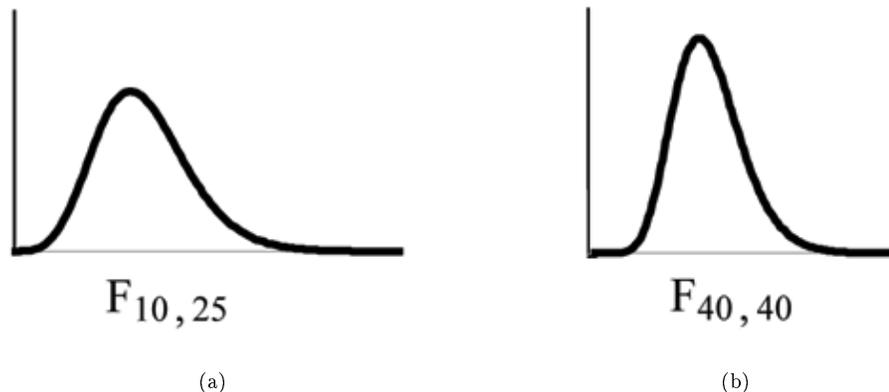


Figure 1

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Example 1

One-Way ANOVA: Four sororities took a random sample of sisters regarding their grade averages for the past term. The results are shown below:

GRADE AVERAGES FOR FOUR SORORITIES			
Sorority 1	Sorority 2	Sorority 3	Sorority 4
2.17	2.63	2.63	3.79
1.85	1.77	3.78	3.45
2.83	3.25	4.00	3.08
1.69	1.86	2.55	2.26
3.33	2.21	2.45	3.18

Table 1

Problem

Using a significance level of 1%, is there a difference in grade averages among the sororities?

Solution

Let $\mu_1, \mu_2, \mu_3, \mu_4$ be the population means of the sororities. Remember that the null hypothesis claims that the sorority groups are from the same normal distribution. The alternate hypothesis says that at least two of the sorority groups come from populations with different normal distributions. Notice that the four sample sizes are each size 5.

$H_o : \mu_1 = \mu_2 = \mu_3 = \mu_4$

H_a : Not all of the means $\mu_1, \mu_2, \mu_3, \mu_4$ are equal.

Distribution for the test: $F_{3,16}$

where $k = 4$ groups and $N = 20$ samples in total

$df(\text{num}) = k - 1 = 4 - 1 = 3$

$df(\text{denom}) = N - k = 20 - 4 = 16$

Calculate the test statistic: $F = 2.23$

Graph:

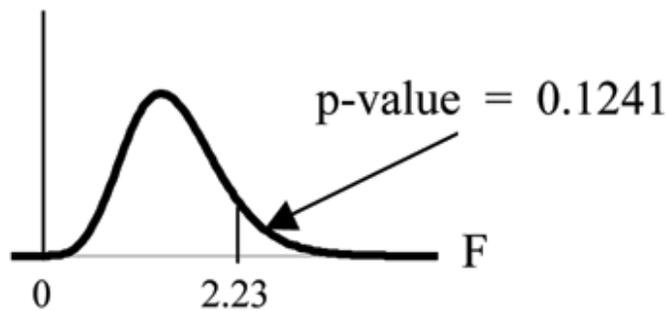


Figure 2



Probability statement: $p\text{-value} = P(F > 2.23) = 0.1241$

Compare α and the p – value: $\alpha = 0.01$ $p\text{-value} = 0.1242$ $\alpha < p\text{-value}$

Make a decision: Since $\alpha < p\text{-value}$, you cannot reject H_0 .

This means that the population averages appear to be the same.

Conclusion: There is not sufficient evidence to conclude that there is a difference among the grade averages for the sororities.

TI-83+ or TI 84: Put the data into lists L1, L2, L3, and L4. Press STAT and arrow over to TESTS. Arrow down to F:ANOVA. Press ENTER and Enter (L1,L2,L3,L4). The F statistic is 2.2303 and the p-value is 0.1241. $df(\text{numerator}) = 3$ (under "Factor") and $df(\text{denominator}) = 16$ (under Error).

Example 2

A fourth grade class is studying the environment. One of the assignments is to grow bean plants in different soils. Tommy chose to grow his bean plants in soil found outside his classroom mixed with dryer lint. Tara chose to grow her bean plants in potting soil bought at the local nursery. Nick chose to grow his bean plants in soil from his mother’s garden. No chemicals were used on the plants, only water. They were grown inside the classroom next to a large window. Each child grew 5 plants. At the end of the growing period, each plant was measured, producing the following data (in inches):

Tommy’s Plants	Tara’s Plants	Nick’s Plants
24	25	23
21	31	27
23	23	22
30	20	30
23	28	20

Table 2

Problem 1

Does it appear that the three media in which the bean plants were grown produce the same average height? Test at a 3% level of significance.

Solution

This time, we will perform the calculations that lead to the F’ statistic. Notice that each group has the same number of plants so we will use the formula $F' = \frac{n \cdot (s-x)^2}{(s_{pooled})}$.

First, calculate the sample mean and sample variance of each group.

	Tommy’s Plants	Tara’s Plants	Nick’s Plants
Sample Mean	24.2	25.4	24.4
Sample Variance	11.7	18.3	16.3

Table 3



Next, calculate the variance of the three group means (Calculate the variance of 24.2, 25.4, and 24.4). **Variance of the group means = 0.413** = $(s_{-x})^2$

Then $MS_{\text{between}} = n(s_{-x})^2 = (5)(0.413)$ where $n = 5$ is the sample size (number of plants each child grew).

Calculate the average of the three sample variances (Calculate the average of 11.7, 18.3, and 16.3). **Average of the sample variances = 15.433** = $(s_{\text{pooled}})^2$

Then $MS_{\text{within}} = (s_{\text{pooled}})^2 = 15.433$.

The F statistic (or F ratio) is $F = \frac{MS_{\text{between}}}{MS_{\text{within}}} = \frac{n \cdot (s_{-x})^2}{(s_{\text{pooled}})^2} = \frac{(5) \cdot (0.413)}{15.433} = 0.134$

The dfs for the numerator = the number of groups $- 1 = 3 - 1 = 2$

The dfs for the denominator = the total number of samples $-$ the number of groups = $15 - 3 = 12$

The distribution for the test is $F_{2,12}$ and the F statistic is $F = 0.134$

The p-value is $P(F > 0.134) = 0.8759$.

Decision: Since $\alpha = 0.03$ and the p-value = 0.8759, do not reject H_o . (Why?)

Conclusion: With a 3% the level of significance, from the sample data, the evidence is not sufficient to conclude that the average heights of the bean plants are not different. Of the three media tested, it appears that it does not matter which one the bean plants are grown in.

(This experiment was actually done by three classmates of the son of one of the authors.)

Another fourth grader also grew bean plants but this time in a jelly-like mass. The heights were (in inches) 24, 28, 25, 30, and 32.

Problem 2

(Solution on p. 5.)

Do an ANOVA test on the 4 groups. You may use your calculator or computer to perform the test. Are the heights of the bean plants different? Use a solution sheet¹.

1 Optional Classroom Activity

Randomly divide the class into four groups of the same size. Have each member of each group record the number of states in the United States he or she has visited. Run an ANOVA test to determine if the average number of states visited in the four groups are the same. Test at a 1% level of significance. Use one of the solution sheets² at the end of the chapter (after the homework).

¹"Collaborative Statistics: Solution Sheets: F Distribution and ANOVA" <<http://cnx.org/content/m17135/latest/>>

²"Collaborative Statistics: Solution Sheets: F Distribution and ANOVA" <<http://cnx.org/content/m17135/latest/>>

Solutions to Exercises in this Module

Solution to Example 2, Problem 2 (p. 4)

- $F = 0.9496$
- $p\text{-value} = 0.4401$

The heights of the bean plants are the same.

F DISTRIBUTION AND ANOVA: HOMEWORK*

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Abstract

This module provides a homework of F Distribution and ANOVA as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

DIRECTIONS: Use a solution sheet to conduct the following hypothesis tests. The solution sheet can be found in the Table of Contents 14. Appendix.

Exercise 1

(Solution on p. 4.)

Three students, Linda, Tuan, and Javier, are given 5 laboratory rats each for a nutritional experiment. Each rat's weight is recorded in grams. Linda feeds her rats Formula A, Tuan feeds his rats Formula B, and Javier feeds his rats Formula C. At the end of a specified time period, each rat is weighed again and the net gain in grams is recorded. Using a significance level of 10%, test the hypothesis that the three formulas produce the same average weight gain.

Weights of Student Lab Rats

Linda's rats	Tuan's rats	Javier's rats
43.5	47.0	51.2
39.4	40.5	40.9
41.3	38.9	37.9
46.0	46.3	45.0
38.2	44.2	48.6

Table 1

Exercise 2

A grassroots group opposed to a proposed increase in the gas tax claimed that the increase would hurt working-class people the most, since they commute the farthest to work. Suppose that the group randomly surveyed 24 individuals and asked them their daily one-way commuting mileage. The results are below:

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working-class	professional (middle incomes)	professional (wealthy)
17.8	16.5	8.5
26.7	17.4	6.3
49.4	22.0	4.6
9.4	7.4	12.6
65.4	9.4	11.0
47.1	2.1	28.6
19.5	6.4	15.4
51.2	13.9	9.3

Table 2

Exercise 3 *(Solution on p. 4.)*

Refer to Exercise 13.8.1. Determine whether or not the variance in weight gain is statistically the same among Javier’s and Linda’s rats.

Exercise 4

Refer to Exercise 13.8.2 above (Exercise 2). Determine whether or not the variance in mileage driven is statistically the same among the working class and professional (middle income) groups.

For the next two problems, refer to the data from Terri Vogel’s Log Book [link pending].

Exercise 5 *(Solution on p. 4.)*

Examine the 7 practice laps. Determine whether the average lap time is statistically the same for the 7 practice laps, or if there is at least one lap that has a different average time from the others.

Exercise 6

Examine practice laps 3 and 4. Determine whether or not the variance in lap time is statistically the same for those practice laps.

For the next four problems, refer to the following data.

The following table lists the number of pages in four different types of magazines.

home decorating	news	health	computer
172	87	82	104
286	94	153	136
163	123	87	98
205	106	103	207
197	101	96	146

Table 3

Exercise 7 *(Solution on p. 4.)*

Using a significance level of 5%, test the hypothesis that the four magazine types have the same average length.

Exercise 8

Eliminate one magazine type that you now feel has an average length different than the others. Redo the hypothesis test, testing that the remaining three averages are statistically the same. Use



a new solution sheet. Based on this test, are the average lengths for the remaining three magazines statistically the same?

Exercise 9

Which two magazine types do you think have the same variance in length?

Exercise 10

Which two magazine types do you think have different variances in length?

Solutions to Exercises in this Module

Solution to Exercise 1 (p. 1)

- a. $H_o: \mu_L = \mu_T = \mu_J$
- c. $df(n) = 2; df(d) = 12$
- e. 0.67
- f. 0.5305
- h. Decision: Do not reject null; Conclusion: Means are same

Solution to Exercise 3 (p. 2)

- c. $df(n) = 4; df(d) = 4$
- e. 3.00
- f. $2(0.1563) = 0.3126$
- h. Decision: Do not reject null; Conclusion: Variances are same

Solution to Exercise 5 (p. 2)

- c. $df(n) = 6; df(d) = 98$
- e. 1.69
- f. 0.1319
- h. Decision: Do not reject null; Conclusion: Average lap times are the same

Solution to Exercise 7 (p. 2)

- a. $H_o: \mu_d = \mu_n = \mu_h = \mu_c$
- b. At least one average is different
- c. $df(n) = 3; df(d) = 16$
- e. 8.69
- f. 0.0012
- h. Decision: Reject null; Conclusion: At least one average is different

F DISTRIBUTION AND ANOVA: PRACTICE*

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Abstract

This module provides a practice on F Distribution and ANOVA as a part of Collaborative Statistics collection (col10522) by Barbara Illowsky and Susan Dean.

1 Student Learning Outcome

- The student will explore the properties of ANOVA.

2 Given

Suppose a group is interested in determining whether teenagers obtain their drivers licenses at approximately the same average age across the country. Suppose that the following data are randomly collected from five teenagers in each region of the country. The numbers represent the age at which teenagers obtained their drivers licenses.

	Northeast	South	West	Central	East
	16.3	16.9	16.4	16.2	17.1
	16.1	16.5	16.5	16.6	17.2
	16.4	16.4	16.6	16.5	16.6
	16.5	16.2	16.1	16.4	16.8
$\bar{x} =$	_____	_____	_____	_____	_____
$s^2 =$	_____	_____	_____	_____	_____

Table 1

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3 Hypothesis

Exercise 1

State the hypotheses.

H_o :

H_a :

4 Data Entry

Enter the data into your calculator or computer.

Exercise 2

degrees of freedom - numerator: $df(n) =$

(Solution on p. 3.)

Exercise 3

degrees of freedom - denominator: $df(d) =$

(Solution on p. 3.)

Exercise 4

F test statistic =

(Solution on p. 3.)

Exercise 5

p-value =

(Solution on p. 3.)

5 Decisions and Conclusions

State the decisions and conclusions (in complete sentences) for the following preconceived levels of α .

Exercise 6

$\alpha = 0.05$

Decision:

Conclusion:

Exercise 7

$\alpha = 0.01$

Decision:

Conclusion:

Solutions to Exercises in this Module

Solution to Exercise 2 (p. 2)

$$df(1) = 4$$

Solution to Exercise 3 (p. 2)

$$df(2) = 15$$

Solution to Exercise 4 (p. 2)

$$\text{Test statistic} = F = 4.22$$

Solution to Exercise 5 (p. 2)

$$0.017$$



F DISTRIBUTION AND ANOVA: TEST OF TWO VARIANCES*

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Abstract

This module provides the assumptions to be considered in order to calculate a Test of Two Variances and how to execute the Test of Two Variances. An example is provided to help clarify the concept.

Another of the uses of the F distribution is testing two variances. It is often desirable to compare two variances rather than two averages. For instance, college administrators would like two college professors grading exams to have the same variation in their grading. In order for a lid to fit a container, the variation in the lid and the container should be the same. A supermarket might be interested in the variability of check-out times for two checkers.

In order to perform a F test of two variances, it is important that the following are true:

1. The populations from which the two samples are drawn are normally distributed.
2. The two populations are independent of each other.

Suppose we sample randomly from two independent normal populations. Let σ_1^2 and σ_2^2 be the population variances and s_1^2 and s_2^2 be the sample variances. Let the sample sizes be n_1 and n_2 . Since we are interested in comparing the two sample variances, we use the F ratio

$$F = \frac{\left[\frac{(s_1)^2}{(\sigma_1)^2} \right]}{\left[\frac{(s_2)^2}{(\sigma_2)^2} \right]}$$

F has the distribution $F \sim F(n_1 - 1, n_2 - 1)$

where $n_1 - 1$ are the degrees of freedom for the numerator and $n_2 - 1$ are the degrees of freedom for the denominator.

If the null hypothesis is $\sigma_1^2 = \sigma_2^2$, then the F-Ratio becomes $F = \frac{\left[\frac{(s_1)^2}{(\sigma_1)^2} \right]}{\left[\frac{(s_2)^2}{(\sigma_2)^2} \right]} = \frac{(s_1)^2}{(s_2)^2}$.

If the two populations have equal variances, then s_1^2 and s_2^2 are close in value and $F = \frac{(s_1)^2}{(s_2)^2}$ is close to 1. But if the two population variances are very different, s_1^2 and s_2^2 tend to be very different, too. Choosing s_1^2 as the larger sample variance causes the ratio $\frac{(s_1)^2}{(s_2)^2}$ to be greater than 1. If s_1^2 and s_2^2 are far apart, then $F = \frac{(s_1)^2}{(s_2)^2}$ is a large number.

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Therefore, if F is close to 1, the evidence favors the null hypothesis (the two population variances are equal). But if F is much larger than 1, then the evidence is against the null hypothesis.

A test of two variances may be left, right, or two-tailed.

Example 1

Two college instructors are interested in whether or not there is any variation in the way they grade math exams. They each grade the same set of 30 exams. The first instructor’s grades have a variance of 52.3. The second instructor’s grades have a variance of 89.9.

Problem

Test the claim that the first instructor’s variance is smaller. (In most colleges, it is desirable for the variances of exam grades to be nearly the same among instructors.) The level of significance is 10%.

Solution

Let 1 and 2 be the subscripts that indicate the first and second instructor, respectively.

$$n_1 = n_2 = 30.$$

$$H_o: \sigma_1^2 = \sigma_2^2 \text{ and } H_a: \sigma_1^2 < \sigma_2^2$$

Calculate the test statistic: By the null hypothesis ($\sigma_1^2 = \sigma_2^2$), the F statistic is

$$F = \frac{\left[\frac{(s_1)^2}{(\sigma_1)^2} \right]}{\left[\frac{(s_2)^2}{(\sigma_2)^2} \right]} = \frac{(s_1)^2}{(s_2)^2} = \frac{52.3}{89.9} = 0.6$$

Distribution for the test: $F_{29,29}$ where $n_1 - 1 = 29$ and $n_2 - 1 = 29$.

Graph: This test is left tailed.

Draw the graph labeling and shading appropriately.

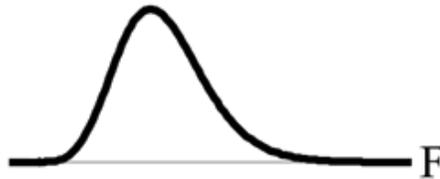


Figure 1

Probability statement: $p\text{-value} = P(F < 0.582) = 0.0755$

Compare α and the p-value: $\alpha = 0.10$ $\alpha > p\text{-value}$.

Make a decision: Since $\alpha > p\text{-value}$, reject H_o .

Conclusion: With a 10% level of significance, from the data, there is sufficient evidence to conclude that the variance in grades for the first instructor is smaller.

TI-83+ and TI-84: Press STAT and arrow over to TESTS. Arrow down to D:2-SampFTest. Press ENTER. Arrow to Stats and press ENTER. For Sx1, n1, Sx2, and n2, enter $\sqrt{52.3}$, 30, $\sqrt{89.9}$, and 30. Press ENTER after each. Arrow to σ_1 : and $<\sigma_2$. Press ENTER. Arrow down to



Calculate and press ENTER. $F = 0.5818$ and p-value = 0.0753. Do the procedure again and try Draw instead of Calculate.

