

Proportional and Nonproportional Relationships

Proportional Relationships

If a linear equation can be written in the form of

$$y = kx$$

Then the relationship is **PROPORTIONAL**.

Based on the form described above, **PROPORTIONAL RELATIONSHIPS vary directly**.

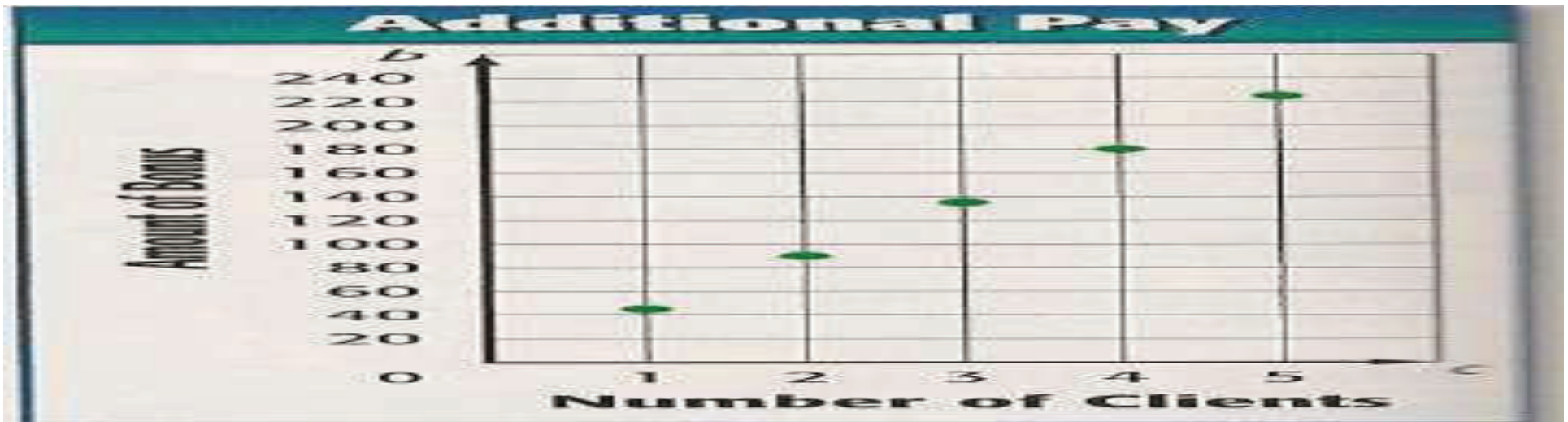
Example 1 – Proportional Relationships

Marcos is a personal trainer at a gym. In addition to his salary, he receives a bonus for each client he sees.

Number of Clients	x	1	2	3	4	5
Bonus Pay (\$)	y	45	90	135	180	225

First, graph this relation and describe any pattern between the number of clients and the amount of bonus pay. Then, write an equation to describe this relationship:

So we have...



The graph demonstrates a linear relationship and it also passes through the point $(0, 0)$ because when Marcos sees 0 clients, he receives 0 pay.

Example 1 - Continued

Now that we have seen that the graph of the Proportional Relationship is linear, we need to calculate the slope – the rate of change – so we have...

$$\text{rate of change} = \frac{\text{bonus pay}}{\# \text{ of clients}}$$

Remember, you can use any two of the given points...

$$\text{rate of change} = \frac{90 - 45}{2 - 1} = \frac{45}{1}$$

So for each client that Marcos sees, he earns \$45 in bonus pay.

Example 1 - Continued

Since we know that **PROPORTIONAL RELATIONSHIPS** vary directly, we know that...

$$y = kx$$

Where, for this problem $k = \frac{45}{1} = 45...$ so we have...

$$y = 45x$$

We can use this equation to find the amount of bonus pay, y , that Marcos will receive for seeing ANY number of clients, x .

You Try 1

A professional soccer team is donating money to a local charity for each goal they score. Write an equation to describe this relationship. Then use this equation to predict how much money will be donated for 12 goals.

Number of Goals	x	1	2	3	4	5
Donations (\$)	y	75	150	225	300	375

If you were correct, you should have found the equation to be...

$$y = 75x$$

And that if the soccer team scores 12 goals, then they will have donated \$900.

Proportional vs. Non-Proportional Relationships

As we should have noticed that **PROPORTIONAL RELATIONSHIPS** vary directly...

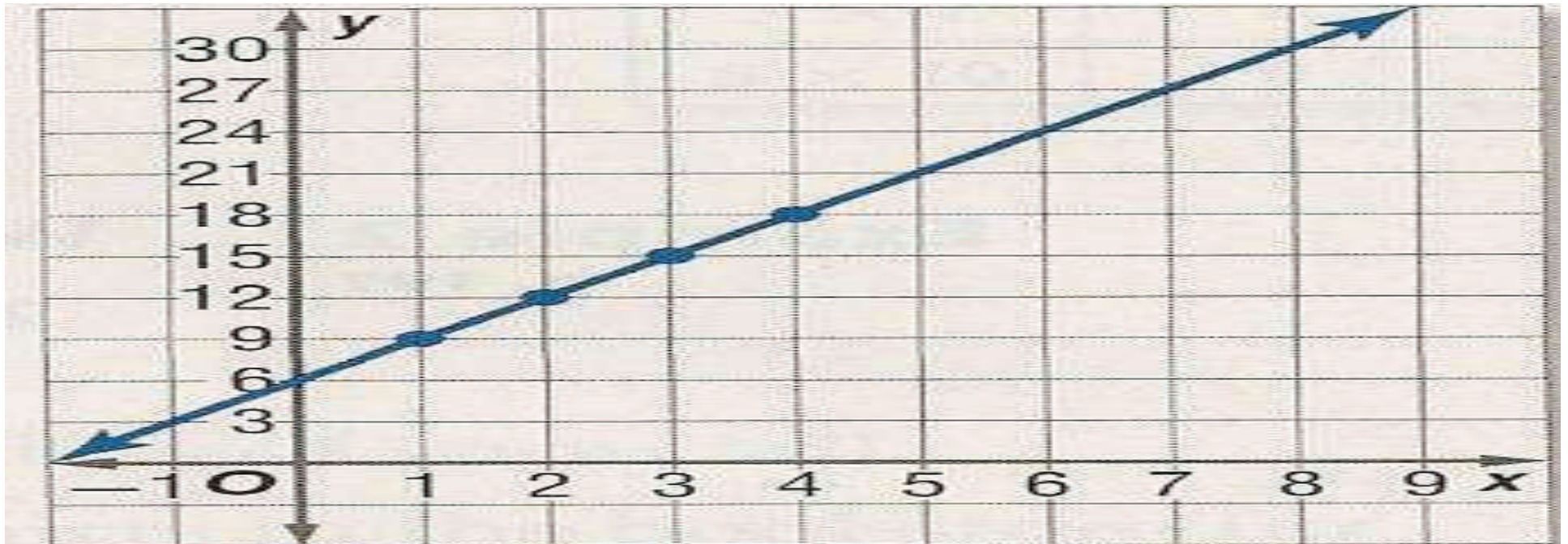
This simply means that for **PROPORTIONAL RELATIONSHIPS**, the graph will **ALWAYS** go through the origin, **(0, 0)**.

On the other hand, **NON-PROPORTIONAL RELATIONSHIPS**, do **NOT** vary directly...

This means that for **NON-PROPORTIONAL RELATIONSHIPS**, the graph will **NOT** go through the origin, **(0, 0)**.

Example 2 – Non-proportional Relationships

Write an equation in function notation for the given graph:



First, we should probably construct a table for a few of the points on the graph...

x	1	2	3	4
y	9	12	15	18

Now we can use the table to find the RATE OF CHANGE...

Example 2 - Continued

We should have that the rate of change is...

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Remember, we can use any two of the given points...

$$m = \frac{12 - 9}{2 - 1} = \frac{3}{1} = 3$$

So the rate of change (or the slope) of the line is 3.

Now, since we know that **NON-PROPORTIONAL RELATIONSHIPS** do **NOT** vary directly, we **CANNOT** use...

$$y = kx$$

Example 2 - Continued

Moreover, we know that **NON-PROPORTIONAL RELATIONSHIPS** do **NOT** pass through – their **Y-Intercepts** are not – the origin, $(0, 0)$.

Instead, we know that the graph passes through – has a **Y-Intercept** of – $(0, 6)$.

So our equation is...

$$y = 3x + 6$$

So In **FUNCTION NOTATION**, our equation is...

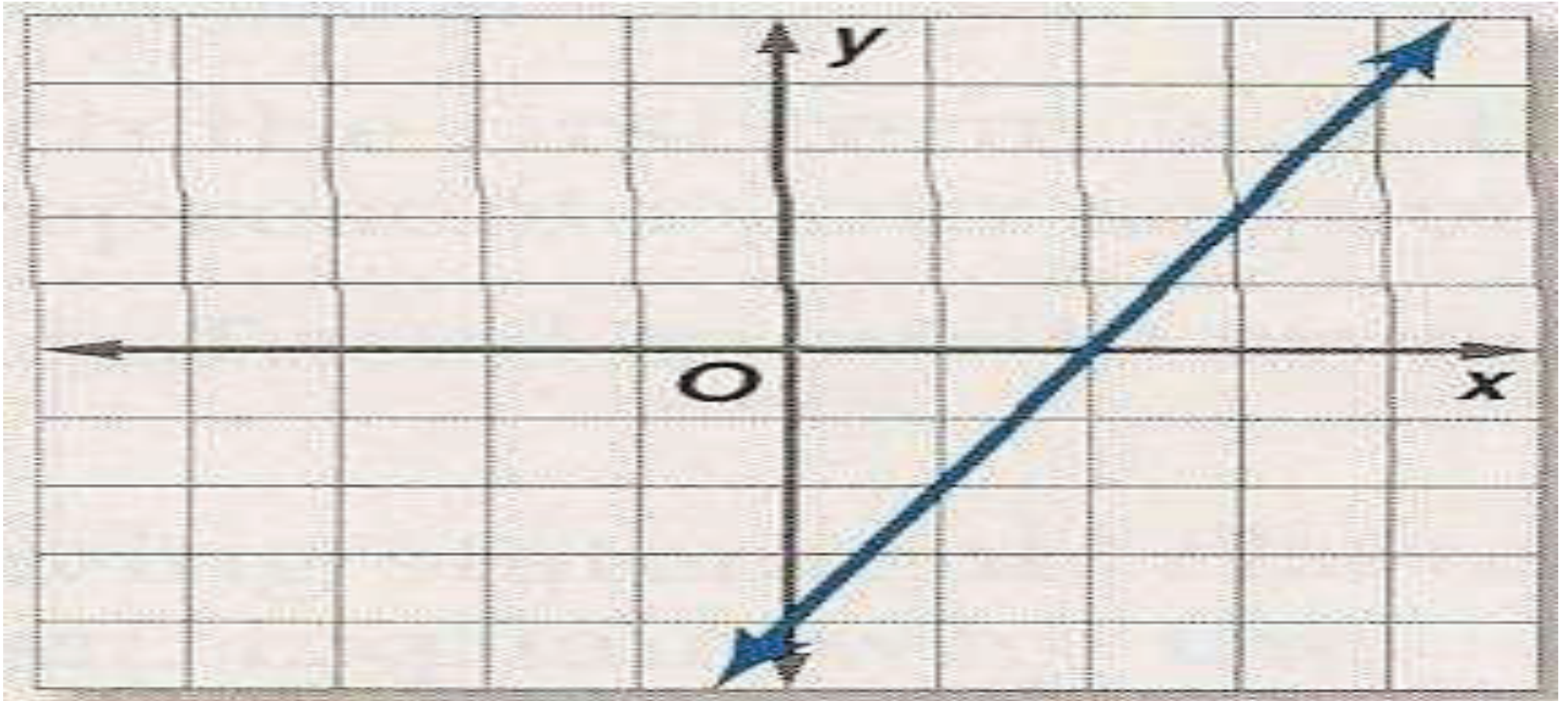
$$f(x) = 3x + 6$$

We can verify this by using the table to check points...

x	1	2	3	4
$3x + 6$	$3(1) + 6$	$3(2) + 6$	$3(3) + 6$	$3(4) + 6$
y	9	12	15	18

You Try 2

Write an equation in function notation for the graph:



If you were correct, you should have found that...

$$y = 2x - 4$$