SE Right Triangle Trigonometry - TI

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Printed: September 23, 2013

flexbook
next generation textbooks

CHAPTER 1 SE Right Triangle Trigonometry - TI

CHAPTER OUTLINE

The activities below are intended to supplement our Geometry flexbooks.

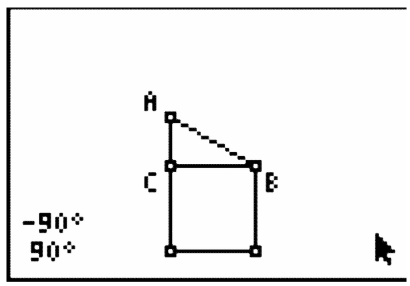
- Geometry, first edition, Chapter 8: http://www.ck12.org/flexr/chapter/2271
- Geometry, second edition, Chapter 8: http://www.ck12.org/flexr/chapter/9416
- Basic Geometry, Chapter 8: http://www.ck12.org/flexr/flexbook/8938

1.1 The Pythagorean Theorem

This activity is intended to supplement Geometry, Chapter 8, Lesson 1.

Problem 1 – Squares on Sides Proof

1. Why is the constructed quadrilateral a square?



2. Record three sets of area measurements you made by dragging points A, B, and/or C.

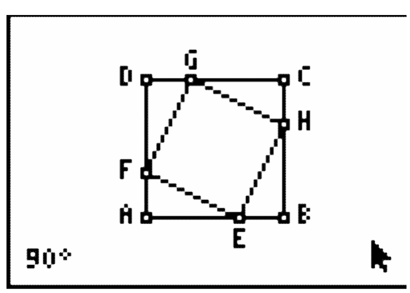
TABLE 1.1:

Square on $\overline{BC}$	**Square on $\overline{AC}$**	**Square on $\overline{AB}$**	**Sum of squares**

3. What conjecture can you make about the areas of the three squares? Does this relationship always hold when a vertex of $\triangle ABC$ is dragged to a different location?

Problem 2 – Inside a Square Proof

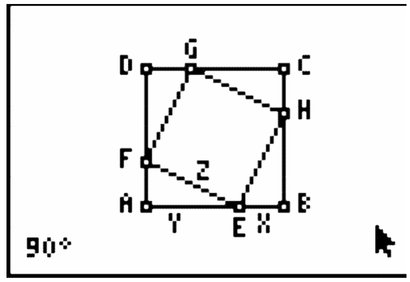
4. Prove that constructed quadrilateral $EFGH$ is a square.



5. $ABCD$ is a square with all sides of length $(x+y)$.

The area of the square $ABCD$ is $(x+y)^2 = x^2 + 2xy + y^2$

Each of the triangles, $\triangle EFA$, $\triangle FGD$, $\triangle GHC$ and $\triangle HEB$, is a right triangle with height x and base y. So, the area of each triangle is $\frac{1}{2}xy$.



$EFGH$ is a square with sides of length z. So the area of $EFGH$ is z^2.

Looking at the areas in the diagram we can conclude that:

$$ABCD = \triangle EFA + \triangle FGD + \triangle GHC + \triangle HEB + EFGH$$

Substitute the area expressions (with variables x, y, and z) into the equation above and simplify.

6. Record three sets of numeric values for $\triangle HEB$.

TABLE 1.2:

BE	BE^2	HB	HB^2	BE^2+HB^2	EH	EH^2

7. Does $BE^2+HB^2=EH^2$ when E is dragged to a different locations?

8. Does $BE^2+HB^2=EH^2$ when A or B are dragged to different locations?

1.2 Investigating Special Triangles

This activity is intended to supplement Geometry, Chapter 8, Lesson 4.

Problem 1 – Investigation of

First, turn on your TI-84 and press **APPS**. Arrow down until you see **Cabri Jr** and press **ENTER**. Open the file **ISOSC**. This file has a triangle with an isosceles triangle with $AB = AC$.

Using the **Perpendicular** tool (**ZOOM** >**Perp.**), construct a perpendicular from point A to side BC. Label the point of intersection of this line with BC as D. To name the point, they need to select the **Alph-Num** tool (**GRAPH** >**Alph-Num**), select the point, and press x^{-1}**ENTER** for the letter D.

Construct line segments BD and CD (π >**Segment**) and then measure the segments (**GRAPH** >**Measure** >**D. Length**).

$BD =$ ________ $CD =$ ________

Would you have expected these segments to be equal in length?

Drag point C to see the effect on the lengths of the line segments. It appears that the perpendicular from the vertex always bisects the opposite side. Measure the angles BAD and CAD.

$\angle BAD =$ ________ $\angle CAD =$ ________

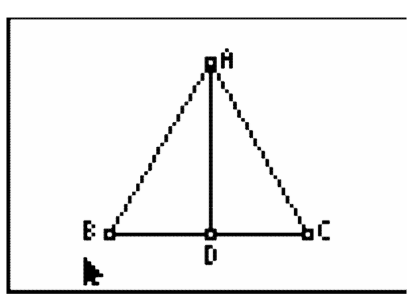
Will they always be equal? ______________________________

Problem 2 – Investigation of

Open the file **EQUIL**. Note that all three angles are 60° angles.

Construct the perpendicular from A to side BC. Label the point of intersection as D.

From the construction above, we know that D bisects BC and that $m\angle BAD = 30°$.



Construct segment BD. We now have triangle BAD where $m\angle D = 90^\circ$, $m\angle B = 60^\circ$ and $m\angle A = 30^\circ$. We also have triangle ACD where $m\angle A = 30^\circ$, $m\angle C = 60^\circ$ and $m\angle D = 90^\circ$.

This completes the construction of two $30^\circ - 60^\circ - 90^\circ$ triangles. We will work only with the triangle BAD.

Measure the three sides of triangle BAD.

$AB =$ ________ $BD =$ ________ $AD =$ ________

Press σ and select the **Calculate** tool. Click on the length of BD, then on the length of AB. Press the ∞ key. Move it to the upper corner. Repeat this step to find the ratio of $AD : AB$ and $AD : BD$. These ratios will become important when you start working with trigonometry.

$BD : AB =$ ________ $AD : AB =$ ________ $AD : BD =$ ________

Drag point C to another location. What do you notice about the three ratios?

Problem 3 – Investigation of

Press the o button and select **New** to open a new document.

To begin the construction of the $45^\circ - 45^\circ - 90^\circ$ triangle, construct line segment AB and a perpendicular to AB at A.

Use the compass tool with center A and radius AB. The circle will intersect the perpendicular line at C.

Hide the circle and construct segments AC and BC.

Explain why $AB = AC$ and why angle $ACB =$ angle ABC?

Why are these two angles 45° each?

Measure the sides of the triangle.

$AC =$ ________ $BC =$ ________ $AB =$ ________

Use the **Calculate** tool to find the ratio of $AC : BC$ and $AC : AB$. Once again, these ratios will be important when you study trigonometry.

Drag point B and observe what happens to the sides and ratios.

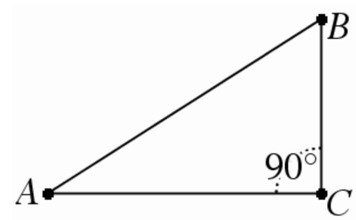
Why do the ratios remain constant while the sides change?

1.3 Ratios of Right Triangles

This activity is intended to supplement Geometry, Chapter 8, Lesson 5.

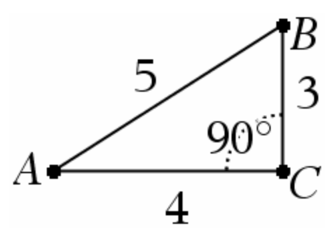
Problem 1 – Exploring Right Triangle Trigonometry

We will begin this activity by looking at the definitions of the sine, cosine, and tangent of a right triangle. Start the *Learning Check* application by pressing **APPS** and selecting **LearnChk**. Open the file *Right Triangle Trigonometry*. You are given the definition for the sine, cosine, and tangent of a right triangle. Copy the definitions onto your worksheet.



1. What is the definition of $\sin A$ for right $\triangle ABC$?
2. What is the definition of $\cos A$ for right $\triangle ABC$?
3. What is the definition of $\tan A$ for right $\triangle ABC$?

Answer the following questions about sine, cosine, and tangent for $\triangle ABC$.



4. What is $\sin A$?
5. What is $\cos A$?
6. What is $\tan A$?
7. What is $\sin B$?
8. What is $\cos B$?
9. What is $\tan B$?

Problem 2 – Exploring the Sine Ratio of a Right Triangle

For this problem, we will investigate the sine ratio. Start the *Cabri Jr.* application by pressing A and selecting **CabriJr**. Open the file *TRIG* by pressing $Y =$, selecting **Open...**, and selecting the file. You are given right triangle ABC.

10. Grab and drag point B. Record the data you collected in the table on the next page. Leave the last column blank for now.

TABLE 1.3:

Position	BC	AB	$\frac{BC}{AB}$	$\sin^{-1}\frac{BC}{AB}$
1				
2				
3				
4				

11. What do you notice about the ratio of BC to AB?

12. Did $\angle A$ change when you moved point B in $\triangle ABC$?

Because the ratio remains the same and $\angle A$ remains fixed, we can use the ratio of BC to AB to find the measurement of $\angle A$. To do this, we will use the definition of sine and the inverse of sine. By definition, $\sin A = \frac{BC}{AB}$. To find the measurement of $\angle A$, we use the inverse of sine to get the formula $A = \sin^{-1}\frac{BC}{AB}$. Exit *Cabri Jr.* and go to the home screen to find the inverse sine of $\frac{BC}{AB}$. Record this into the last column of the table above.

13. What is the measurement of $\angle A$?

14. What is the measurement of $\angle B$?

Problem 3 – Exploring the Cosine Ratio of a Right Triangle

For this problem, we will investigate the sine ratio. Start the *Cabri Jr.* application and open the file *TRIG*. You are given right triangle ABC.

15. Collect data for four positions of point B like that which was done in Problem 2.

TABLE 1.4:

Position	AC	AB	$\frac{AC}{AB}$	$\cos^{-1}\frac{AC}{AB}$
1				
2				
3				
4				

Because the ratio remains the same, and $\angle A$ remains fixed, we can use the ratio of AC to AB to find the measurement of $\angle A$. To do this, we will use the definition of cosine and the inverse of cosine. By definition, $\cos A = \frac{AC}{AB}$. To find the measurement of $\angle A$, we use the inverse of cosine to get the formula $A = \cos^{-1}\frac{AC}{AB}$. Exit *Cabri Jr.* and go to the home screen to find the inverse cosine of $\frac{AC}{AB}$. Record this into the last column of the table above.

16. What is the measurement of $\angle A$?

17. What is the measurement of $\angle B$?

18. How would you solve an equation of the form $\tan A = \frac{BC}{AC}$?

Problem 4 – Applying the Sine, Cosine, and Tangent Ratio of a Right Triangle

Find and label the measure of each angle given two sides of the right triangle.

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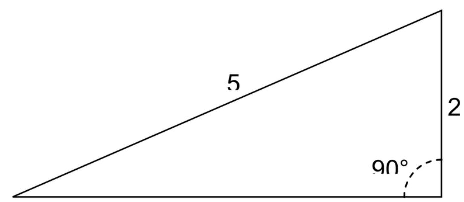

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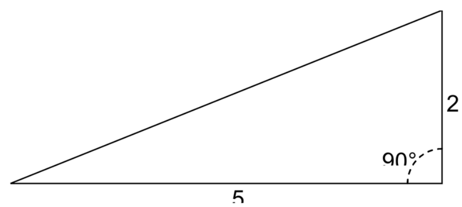

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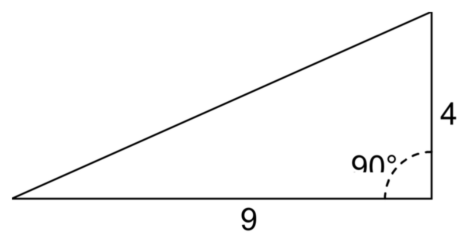

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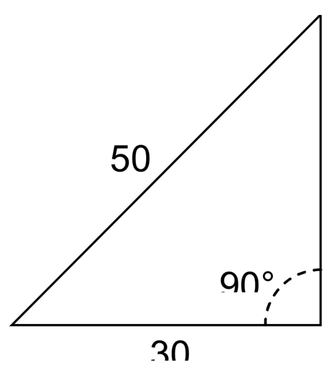

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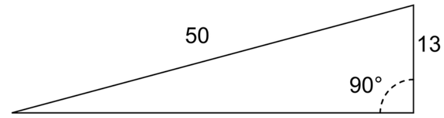

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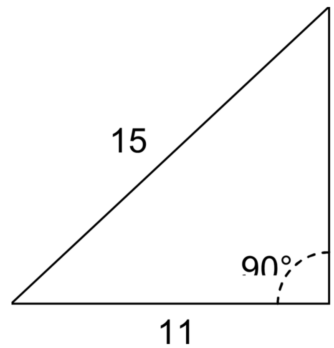

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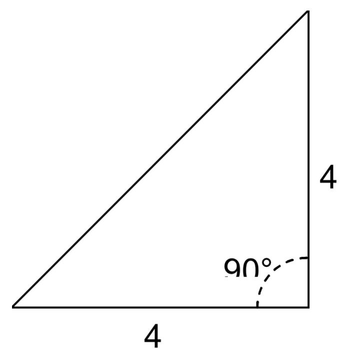

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