Precalculus

An Investigation of Functions

Edition 1.3

David Lippman
Melonie Rasmussen

This book is also available to read free online at
http://www.opentextbookstore.com/precalc/
If you want a printed copy, buying from the bookstore is cheaper than printing yourself.

Selected exercises were remixed from *Precalculus* by D.H. Collingwood and K.D. Prince, originally licensed under the GNU Free Document License, with permission from the authors.

Cover Photo by David Lippman, of artwork by
John Rogers
Lituus, 2010
Dichromatic glass and aluminum
Washington State Arts Commission in partnership with Pierce College

This is the fourth official version of Edition 1. It contains typo corrections and language clarification, but is page number and problem set number equivalent to the original Edition 1.

Section 4.5 Graphs of Logarithmic Functions

Recall that the exponential function $f(x) = 2^x$ produces this table of values

x	-3	-2	-1	0	1	2	3
f(x)	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

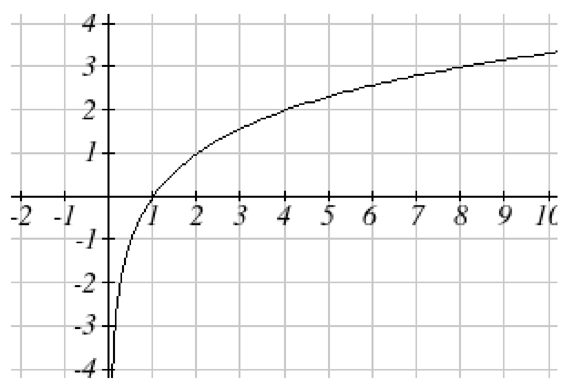
Since the logarithmic function is an inverse of the exponential, $g(x) = \log_2(x)$ produces the table of values

x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
g(x)	-3	-2	-1	0	1	2	3

In this second table, notice that

1) As the input increases, the output increases.
2) As input increases, the output increases more slowly.
3) Since the exponential function only outputs positive values, the logarithm can only accept positive values as inputs, so the domain of the log function is $(0, \infty)$.
4) Since the exponential function can accept all real numbers as inputs, the logarithm can output any real number, so the range is all real numbers or $(-\infty, \infty)$.

Sketching the graph, notice that as the input approaches zero from the right, the output of the function grows very large in the negative direction, indicating a vertical asymptote at $x = 0$.
In symbolic notation we write
as $x \to 0^+, f(x) \to -\infty$, and
as $x \to \infty, f(x) \to \infty$

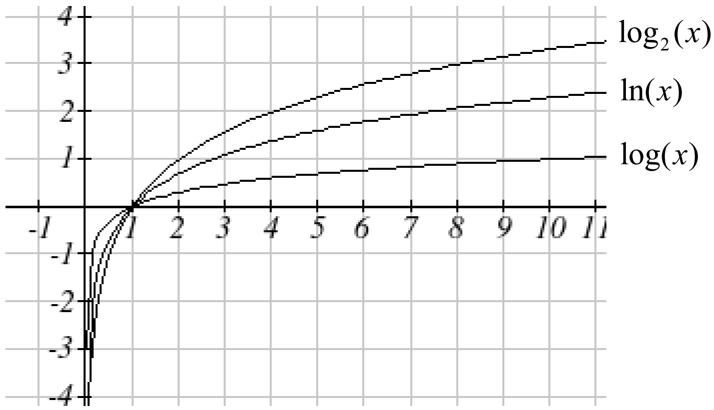


Graphical Features of the Logarithm

Graphically, in the function $g(x) = \log_b(x)$
The graph has a horizontal intercept at (1, 0)
The graph has a vertical asymptote at $x = 0$
The graph is increasing and concave down
The domain of the function is $x > 0$, or $(0, \infty)$
The range of the function is all real numbers, or $(-\infty, \infty)$

When sketching a general logarithm with base *b*, it can be helpful to remember that the graph will pass through the points (1, 0) and (*b*, 1).

To get a feeling for how the base affects the shape of the graph, examine the graphs below.



Notice that the larger the base, the slower the graph grows. For example, the common log graph, while it grows without bound, it does so very slowly. For example, to reach an output of 8, the input must be 100,000,000.

Another important observation made was the domain of the logarithm. Like the reciprocal and square root functions, the logarithm has a restricted domain which must be considered when finding the domain of a composition involving a log.

Example 1

Find the domain of the function $f(x) = \log(5 - 2x)$

The logarithm is only defined with the input is positive, so this function will only be defined when $5 - 2x > 0$. Solving this inequality,

$-2x > -5$

$x < \frac{5}{2}$

The domain of this function is $x < \frac{5}{2}$, or in interval notation, $\left(-\infty, \frac{5}{2}\right)$

Try it Now

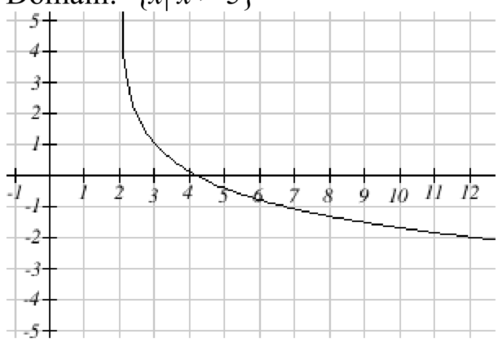
1. Find the domain of the function $f(x) = \log(x - 5) + 2$; before solving this as an inequality, consider how the function has been transformed.

Important Topics of this Section

Graph of the logarithmic function (domain and range)
Transformation of logarithmic functions
Creating graphs from equations
Creating equations from graphs

Try it Now Answers

1. Domain: $\{x \mid x > 5\}$

2. 

Flashback Answers

3. Domain: $\{x|x>-2\}$, Range: all real numbers; As $x \to -2^{+}, f(x) \to \infty$ and as $x \to \infty, f(x) \to -\infty$.