

Precalculus

An Investigation of Functions



Edition 1.3

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This is the fourth official version of Edition 1. It contains typo corrections and language clarification, but is page number and problem set number equivalent to the original Edition 1.

Piecewise Functions

In the toolkit functions we introduced the absolute value function $f(x) = |x|$.

With a domain of all real numbers and a range of values greater than or equal to 0, the absolute value can be defined as the magnitude or modulus of a number, a real number value regardless of sign, the size of the number, or the distance from 0 on the number line. All of these definitions require the output to be greater than or equal to 0.

If we input 0, or a positive value the output is unchanged

$$f(x) = x \quad \text{if } x \geq 0$$

If we input a negative value the sign must change from negative to positive.

$$f(x) = -x \quad \text{if } x < 0 \quad \text{since multiplying a negative value by } -1 \text{ makes it positive.}$$

Since this requires two different processes or pieces, the absolute value function is often called the most basic piecewise defined function.

Piecewise Function

A **piecewise function** is a function in which the formula used depends upon the domain the input lies in. We notate this idea like:

$$f(x) = \begin{cases} \text{formula 1} & \text{if domain to use formula 1} \\ \text{formula 2} & \text{if domain to use formula 2} \\ \text{formula 3} & \text{if domain to use formula 3} \end{cases}$$

Example 5

A museum charges \$5 per person for a guided tour with a group of 1 to 9 people, or a fixed \$50 fee for 10 or more people in the group. Set up a function relating the number of people, n , to the cost, C .

To set up this function, two different formulas would be needed. $C = 5n$ would work for n values under 10, and $C = 50$ would work for values of n ten or greater. Notating this:

$$C(n) = \begin{cases} 5n & \text{if } 0 < n < 10 \\ 50 & \text{if } n \geq 10 \end{cases}$$

Example 6

A cell phone company uses the function below to determine the cost, C , in dollars for g gigabytes of data transfer.

$$C(g) = \begin{cases} 25 & \text{if } 0 < g < 2 \\ 25 + 10(g - 2) & \text{if } g \geq 2 \end{cases}$$

Find the cost of using 1.5 gigabytes of data, and the cost of using 4 gigabytes of data.

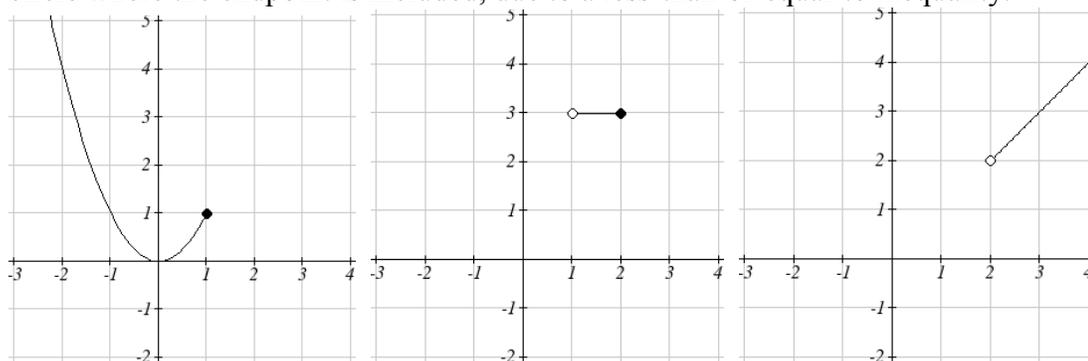
To find the cost of using 1.5 gigabytes of data, $C(1.5)$, we first look to see which piece of domain our input falls in. Since 1.5 is less than 2, we use the first formula, giving $C(1.5) = \$25$.

To find the cost of using 4 gigabytes of data, $C(4)$, we see that our input of 4 is greater than 2, so we'll use the second formula. $C(4) = 25 + 10(4-2) = \$45$.

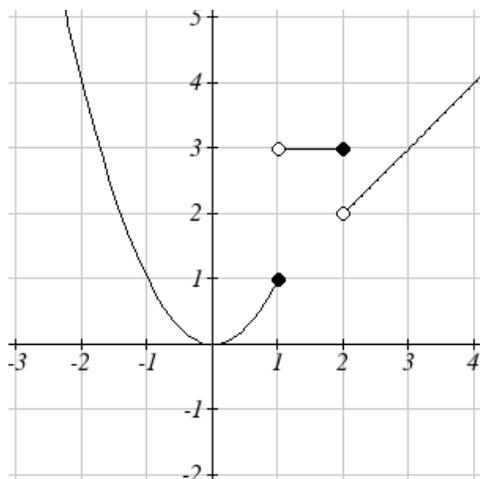
Example 7

Sketch a graph of the function $f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 3 & \text{if } 1 < x \leq 2 \\ x & \text{if } x > 2 \end{cases}$

Since each of the component functions are from our library of Toolkit functions, we know their shapes. We can imagine graphing each function, then limiting the graph to the indicated domain. At the endpoints of the domain, we put open circles to indicate where the endpoint is not included, due to a strictly-less-than inequality, and a closed circle where the endpoint is included, due to a less-than-or-equal-to inequality.



Now that we have each piece individually, we combine them onto the same graph:



Try it Now

4. At Pierce College during the 2009-2010 school year tuition rates for in-state residents were \$89.50 per credit for the first 10 credits, \$33 per credit for credits 11-18, and for over 18 credits the rate is \$73 per credit⁴. Write a piecewise defined function for the total tuition, T , at Pierce College during 2009-2010 as a function of the number of credits taken, c . Be sure to consider a reasonable domain and range.

Important Topics of this Section

Definition of domain
 Definition of range
 Inequalities
 Interval notation
 Set builder notation
 Domain and Range from graphs
 Domain and Range of toolkit functions
 Piecewise defined functions

Try it Now Answers

1. Domain; $y = \text{years}$ [1960,2010] ; Range, $p = \text{population}$, [100,1400]
2. a. Values that are less than or equal to -2, or values that are greater than or equal to -1 and less than 3
 b. $\{x \mid x \leq -2 \text{ or } -1 \leq x < 3\}$
 c. $(-\infty, -2] \cup [-1, 3)$
3. Domain; $y = \text{years}$, [1952,2002] ; Range, $p = \text{population in millions}$, [40,88]

$$4. T(c) = \begin{cases} 89.5c & \text{if } c \leq 10 \\ 895 + 33(c - 10) & \text{if } 10 < c \leq 18 \\ 1159 + 73(c - 18) & \text{if } c > 18 \end{cases} \text{ Tuition, } T, \text{ as a function of credits, } c.$$

Reasonable domain should be whole numbers 0 to (answers may vary), e.g. [0, 23]

Reasonable range should be \$0 – (answers may vary), e.g. [0,1524]

⁴ https://www.pierce.ctc.edu/dist/tuition/ref/files/0910_tuition_rate.pdf, retrieved August 6, 2010