## CHAPTER III.

### SIMPLE BRACKETS. ADDITION.

20. WHEN a number of Arithmetical quantities are connected together by the signs + and −, the value of the result is the same in whatever order the terms are taken. This also holds in the case of Algebraical quantities.

Thus $a-b+c$ is equivalent to $a+c-b$, for in the first of the two expressions $b$ is taken from $a$, and $c$ added to the result; in the second $c$ is added to $a$, and $b$ taken from the result. Similar reasoning applies to all Algebraical expressions. Hence we may write the terms of an expression in any order we please.

Thus it appears that the expression $a-b$ may be written in the equivalent form $-b+a$.

To illustrate this we may suppose, as in Art. 18, that $a$ represents a gain of $a$ pounds, and $-b$ a loss of $b$ pounds: it is clearly immaterial whether the gain precedes the loss, or the loss precedes the gain.

21. A bracket ( ) indicates that the terms enclosed within it are to be considered as one quantity. The full use of brackets will be considered in Chap. VII.; here we shall deal only with the simpler cases.

$8+(13+5)$ means that 13 and 5 are to be added and their sum added to 8. It is clear that 13 and 5 may be added separately or together without altering the result.

Thus $$8+(13+5)=8+13+5=26.$$

Similarly $a+(b+c)$ means that the sum of $b$ and $c$ is to be added to $a$.

Thus $$a+(b+c)=a+b+c.$$

$8+(13-5)$ means that to 8 we are to add the excess of 13 over 5; now if we add 13 to 8 we have added 5 too much, and must therefore take 5 from the result.

Thus $8+(13-5)=8+13-5=16.$

Similarly $a+(b-c)$ means that to $a$ we are to add $b$, diminished by $c$.

Thus $a+(b-c)=a+b-c$ ........................(1).

In like manner,

$$a+b-c+(d-e-f)=a+b-c+d-e-f \text{ .........(2).}$$

Conversely,

$$a+b-c+d-e-f=a+b-c+(d-e-f) \text{ .........(3).}$$

Again, $a-b+c=a+c-b$, [Art. 20.]

$=$the sum of $a$ and $c-b$,

$=$the sum of $a$ and $-b+c$, [Art. 20.]

therefore $a-b+c=a+(-b+c)$ ........................... (4).

By considering the results (1), (2), (3), (4) we are led to the following rule:

RULE. *When an expression within brackets is preceded by the sign +, the brackets can be removed without making any change in the expression.*

Conversely: *Any part of an expression may be enclosed within brackets and the sign + prefixed, the sign of every term within the brackets remaining unaltered.*

Thus the expression $a-b+c-d+e$ may be written in any of the following ways,

$$a+(-b+c-d+e),$$
$$a-b+(c-d+e),$$
$$a-b+c+(-d+e).$$

22. The expression $a-(b+c)$ means that from $a$ we are to take the sum of $b$ and $c$. The result will be the same whether $b$ and $c$ are subtracted separately or in one sum. Thus

$$a-(b+c)=a-b-c.$$

Again, $a-(b-c)$ means that from $a$ we are to subtract the excess of $b$ over $c$. If from $a$ we take $b$ we get $a-b$; but by so doing we shall have taken away $c$ too much, and must therefore add $c$ to $a-b$. Thus

$$a-(b-c)=a-b+c.$$

In like manner,

$$a-b-(c-d-e)=a-b-c+d+e.$$

Accordingly the following rule may be enunciated:

RULE. *When an expression within brackets is preceded by the sign* $-$, *the brackets may be removed if the sign of every term within the brackets be changed.*

Conversely: *Any part of an expression may be enclosed within brackets and the sign* $-$ *prefixed, provided the sign of every term within the brackets be changed.*

Thus the expression $a-b+c+d-e$ may be written in any of the following ways,

$$a-(+b-c-d+e),$$
$$a-b-(-c-d+e),$$
$$a-b+c-(-d+e).$$

## ADDITION.

23. When two or more *like* terms are to be added together we have seen that they may be collected and the result expressed as a single like term. If, however, the terms are *unlike* they cannot be collected. Thus we write the sum of $a$ and $b$ in the form $a+b$.

Also, by the rules for removing brackets, $a+(-b)=a-b$; that is, the sum of $a$ and $-b$ is written in the form $a-b$.

*Example* 1. Find the sum of $x$, $x^2$, $x^3$.

Since different powers of the same letter are unlike terms, the sum is $x+x^2+x^3$.

This expression cannot be abridged.

*Example* 2. The sum of $x$, $-x^2$, $-x^3$, $x^4$ is $x-x^2-x^3+x^4$.

*Example* 3. Find the sum of $3a-5b+2c$ and $2a+3b-c$.

The result may be written $3a-5b+2c+(2a+3b-c)$,

$$=3a-5b+2c+2a+3b-c,$$
$$=3a+2a-5b+3b+2c-c,$$
$$=5a-2b+c,$$

by collecting the like terms.

The operation is more conveniently performed as follows:

$$\begin{array}{r}3a-5b+2c\\ 2a+3b-\phantom{2}c\\ \hline 5a-2b+\phantom{2}c.\end{array}$$

RULE. *Arrange the expressions in lines so that the like terms may be in the same vertical columns: then add each column beginning with that on the left.*

*Example.* Find the sum of $3a-7b+2c$; $6a-b+5c$; $-4a+3b-8c$.

$$\begin{array}{r} 3a-7b+2c \\ 6a-\phantom{7}b+5c \\ -4a+3b-8c \\ \hline 5a-5b-\phantom{2}c. \end{array}$$

24. From the foregoing Examples it will be observed that in Algebra the word *sum* is used in a wider sense than in Arithmetic. Thus, in the language of Arithmetic, $a-b$ signifies that $b$ is to be subtracted from $a$, and bears that meaning only; but in Algebra it is also taken to mean the sum of the two quantities $a$ and $-b$ without any regard to the relative magnitudes of $a$ and $b$.

When quantities are connected by the signs $+$ and $-$, the resulting expression is called their **Algebraical Sum.**

Thus $$11a-27a+13a=-3a$$
states that the Algebraical Sum of $11a$, $-27a$, and $13a$ is equal to $-3a$.

## EXAMPLES III. a.

Find the sum of

1. $a+2b-3c$; $-3a+b+2c$; $2a-3b+c$.
2. $3a+2b-c$; $-a+3b+2c$; $2a-b+3c$.
3. $-3x+2y+z$; $x-3y+2z$; $2x+y-3z$.
4. $-x+2y+3z$; $3x-y+2z$; $2x+3y-z$.
5. $4a+3b+5c$; $-2a+3b-8c$; $a-b+c$.
6. $-15a-19b-18c$; $14a+15b+8c$; $a+5b+9c$.
7. $25a-15b+c$; $13a-10b+4c$; $a+20b-c$.
8. $-16a-10b+5c$; $10a+5b+c$; $6a+5b-c$.
9. $5ax-7by+cz$; $ax+2by-cz$; $-3ax+2by+3cz$.
10. $20p+q-r$; $p-20q+r$; $p+q-20r$.

Add together the following expressions:

11. $-5ab+6bc-7ca$; $8ab-4bc+3ca$; $-2ab-2bc+4ca$.
12. $15ab-27bc-6ca$; $14ab-18bc+10ca$; $-49ab+45bc-3ca$.
13. $5ab+bc-3ca$; $ab-bc+ca$; $-ab+bc+2ca$.
14. $pq+qr-rp$; $-pq+qr+rp$; $pq-qr+rp$.
15. $x+y+z$; $2x+3y-2z$; $3x-4y+z$.

**16.** $2a-3b+c$; $15a-21b-8c$; $3a+24b+7c$.

**17.** $4xy-9yz+2zx$; $-25xy+24yz-zx$; $23xy-15yz+zx$.

**18.** $17ab-13bc+8ca$; $-5ab+9bc-7ca$; $2ab-7bc-ca$.

**19.** $47x-63y+z$; $-25x+15y-3z$; $-22x+48y+15z$.

**20.** $23a-17b-2c$; $-9a+15b+7c$; $-13a+3b-4c$.

25. In adding together several algebraical expressions containing terms with different powers of the same letter, it will be found convenient to arrange all the expressions in *descending* or *ascending* powers of that letter.

*Example* 1. Add together $3x^3+7-5x^2$; $2x^2-8-9x$; $4x-2x^3+3x^2$.

$$\begin{array}{rrrr} 3x^3 & -5x^2 & & +7 \\ & 2x^2 & -9x & -8 \\ -2x^3 & +3x^2 & +4x & \\ \hline x^3 & & -5x & -1. \end{array}$$

The result is in *descending* powers of $x$.

*Example* 2. Add together

$3ab^2-2b^3+a^3$; $5a^2b-ab^2-3a^3$; $8a^3+5b^3$; $9a^2b-2a^3+ab^2$.

$$\begin{array}{rrrr} -2b^3 & +3ab^2 & & +a^3 \\ & -ab^2 & +5a^2b & -3a^3 \\ 5b^3 & & & +8a^3 \\ & ab^2 & +9a^2b & -2a^3 \\ \hline 3b^3 & +3ab^2 & +14a^2b & +4a^3. \end{array}$$

In this example the student should notice that the result is expressed according to *descending* powers of $b$, and *ascending* powers of $a$.

## EXAMPLES III. b.

Find the sum of

**1.** $2ab+3ca+6abc$; $-5ab+2bc-5abc$; $3ab-2bc-3ca$.

**2.** $2x^2-2xy+3y^2$; $4y^2+5xy-2x^2$; $x^2-2xy-6y^2$.

**3.** $3a^2-7ab-4b^2$; $-6a^2+9ab-3b^2$; $4a^2+ab+5b^2$.

**4.** $x^2+xy-y^2$; $-z^2+yz+y^2$; $-x^2+xz+z^2$.

**5.** $-x^2-3xy+3y^2$; $3x^2+4xy-5y^2$; $x^2+xy+y^2$.

6. $x^3-x^2+x-1$; $2x^2-2x+2$; $-3x^3+5x+1$.
7. $2x^3-x^2-x$; $4x^3+8x^2+7x$; $-6x^3-6x^2+x$.
8. $9x^2-7x+5$; $-14x^2+15x-6$; $20x^2-40x-17$.
9. $10x^3+5x+8$; $3x^3-4x^2-6$; $2x^3-2x-3$.
10. $a^3-ab+bc$; $ab+b^3-ca$; $ca-bc+c^3$.
11. $5a^3-3c^3+d^3$; $b^3-2a^3+3d^3$; $4c^3-2a^3-3d^3$.
12. $6x^3-2x+1$; $2x^3+x+6$; $x^2-7x^3+2x-4$.
13. $a^3-a^2+3a$; $3a^3+4a^2+8a$; $5a^3-6a^2-11a$.
14. $x^2+y^2-2xy$; $2z^2-3y^2-4yz$; $2x^2-2z^2-3xz$.
15. $x^3-2y^3+x$; $y^3-2x^3+y$; $x^2+2y^2-x+y^3$.
16. $x^3+3x^2y+3xy^2$; $-3x^2y-6xy^2-x^3$; $3x^2y+4xy^2$.
17. $a^3+5ab^2+b^3$; $b^3-10ab^2-a^3$; $5ab^2-2b^3+2a^2b$.
18. $x^5-4x^4y-5x^3y^3$; $3x^4y+2x^3y^3-6xy^4$; $3x^3y^3+6xy^4-y^5$.
19. $a^3-4a^2b+6abc$; $a^2b-10abc+c^3$; $b^3+3a^2b+abc$.
20. $x^3-4x^2y+6xy^2$; $2x^2y-3xy^2+2y^3$; $y^3+3x^2y+4xy^2$.

Add together the following expressions:

21. $\frac{1}{2}a-\frac{1}{3}b$; $-a+\frac{2}{3}b$; $\frac{3}{4}a-b$.
22. $-\frac{1}{3}a-\frac{1}{4}b$; $-\frac{2}{3}a+\frac{3}{4}b$; $-2a-b$.
23. $-2a+\frac{5}{2}c$; $-\frac{1}{3}a-2b$; $\frac{8}{3}b-3c$.
24. $-\frac{13}{8}a-\frac{11}{4}c$; $2a-3b$; $\frac{11}{5}b-c$.
25. $\frac{2}{3}x^2+\frac{1}{3}xy-\frac{1}{4}y^2$; $-x^2-\frac{2}{3}xy+2y^2$; $\frac{2}{3}x^2-xy-\frac{5}{4}y^2$.
26. $3a^2-\frac{2}{5}ab-\frac{1}{2}b^2$; $-\frac{3}{2}a^2+2ab-\frac{2}{3}b^2$; $-\frac{2}{3}a^2-ab+b^2$.
27. $\frac{5}{8}x^2-\frac{1}{3}xy+\frac{3}{10}y^2$; $-\frac{3}{4}x^2+\frac{14}{15}xy-y^2$; $\frac{1}{2}x^2-xy+\frac{1}{5}y^2$.
28. $-\frac{3}{4}x^3+5ax^2-\frac{5}{8}a^2x$; $x^3-\frac{37}{8}ax^2+\frac{1}{2}a^2x$; $-\frac{1}{2}x^3+\frac{3}{4}a^2x$.
29. $\frac{3}{8}x^2-\frac{5}{3}xy-7y^2$; $\frac{2}{3}xy+\frac{18}{5}y^2$; $-\frac{5}{8}x^2+4y^2$.
30. $\frac{1}{2}a^3-2a^2b-\frac{3}{2}b^3$; $\frac{3}{2}a^2b-\frac{3}{4}ab^2+2b^3$; $-\frac{3}{2}a^3+ab^2+\frac{1}{2}b^3$.