CS202 – Subunit 1.3.1
The Saylor Foundation's "Modus Ponens and Other Types of Reasoning"

Modus Ponens is a rule of inference used in Logic. It is one tool in formulating an argument or proof. Given assumed statements (hypothesis), a rule of inference allows us to derive a conclusion statement using the definitions and properties of the operations and symbols. Modus Ponens is the Latin name for "a way of affirming." (In English) If P is True, then Q is True. P is True. Therefore, Q is True. In symbols, P => Q AND P |- Q. The symbol, '|-', is called a 'turnsile.' A variant is Modus Tollens, which means "a way of denying." (In English) If P is True, then Q is True. Q is false. Therefore, P is False.

In symbols, P => Q AND NOT Q |- NOT P. (Note: Rather than write out P is True, we simply write P.) Modus Ponens is also written with the hypothesis above a line and the conclusion below the line:

$$\frac{P \Rightarrow Q,\ P}{Q}$$

With reference to our day to day experiences (real world domain) or with reference to mathematical statements (a mathematics domain), we often refer to specific instances or refer to a variable that ranges over a set. For example, John is a boy, or 5 is positive. John is a special instance of all boys; with reference to mathematics, 5 is a specific instance of a positive number. For another example consider, Boys in Chicago, or x is a positive number. In these examples, boy ranges over the set of boys in Chicago, and x ranges over the set of positive numbers. If we extend the Propositional Calculus to statements that include variables that take on specific values that range over a set, we get a logic that is known as the Predicate Calculus. The definitions, rules, and properties that apply to the Propositional Calculus also apply to the Predicate Calculus (also called the First Order Predicate Calculus.) We will study the Predicate Calculus in Unit 2 of this course.

In addition to Modus Ponens, other rules of inference in logic include Generalization, Specialization, Elimination, and several others, summarized below.

Generalization is a rule of inference typically used in the Predicate Calculus. It allows us to conclude that for an arbitrary y P(y), then for all x P(x). P(y) is shorthand notation, for P(y) True; similarly, for P(x). This generalizes the statement to a larger set, i.e. all x. Here, x and y are simple variables that represent simple objects in a reference domain (i.e. real world, or a mathematics system). In symbols, P(x) |- for all x P(x). For all x, is also written as:

$$\forall\ x\ P(x).$$

We will examine this more in-depth in Unit 2. In the Propositional Calculus, generalization takes the form of eliminating parts of a statement. See the summary table below.

Specialization is another rule of inference for the Predicate Calculus. Given x P(x), then P(b) for a specific b. In symbols, $\exists$ x P(x) |- P(b)., where b is an instance for which P is True. We also say there exists a 'b,' such that P(b). In the Propositional Calculus specialization takes the form of adding statements. See the summary table below.

Elimination is a rule of inference that applies in the same form to both the propositional and predicate calculus. P /\ Q |- Q or P(x) /\ Q(x) |- Q(x), where x is arbitrary (i.e. for all x). In conclusion, we could replace Q by P. Thus, P /\ Q |- P.

Transitivity also takes the same form for both the Propositional and Predicate Calculus. P = Q is a proposition that is True when both P and Q are True, or when P and Q are both False. The transitivity rule of propositional and predicate calculus states,

$$P = Q, Q = R \ \ |- P = R$$

Please also refer to the summary table in the instructions section of the Wikipedia resource for section 1.3.1.